Hart Gibson Lester.

May 23. 1906.

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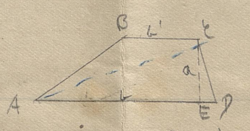
I



ABCD = a trapezoid

To prove ABCD = a(b+b')

Proof Draw ~~altitude~~ altitude a and diagonal AC.

$\triangle ABC = \frac{1}{2} a \times b'$ (the area of a $\triangle$ = ½ the product of its bas × its alt)

$\triangle ACD = \frac{1}{2} a \times b$ (Same ")

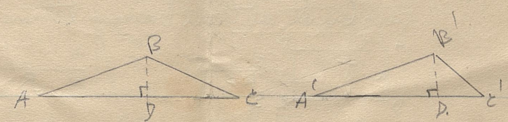
$\triangle ABC + \triangle ACD = \frac{1}{2} a(b+b')$ adding (the whole = the sum of all its parts)

or $ABCD = \frac{1}{2} a(b+b')$

Q E D.

2 D

II



Given $\triangle$ ABC & A'B'C' with $\angle A = \angle A'$.

to prove. $\dfrac{\triangle ABC}{\triangle A'B'C'} = \dfrac{AB \cdot AC}{A'B' \cdot A'C'}$.

Proof. Draw alts. BD and B'D'

$\triangle ABD \sim \triangle A'B'D'$. (2 $\triangle$ are similar if they have 2 $\angle$s of one = respectively to 2 $\angle$s of another)

$\therefore \dfrac{AB}{A'B'} = \dfrac{BD}{B'D'}$ (homo sides of similar $\triangle$s are proportional)

$\dfrac{\triangle ABC}{\triangle A'B'C'} = \dfrac{BD \cdot AC}{B'D' \cdot A'C'}$ (2 $\triangle$ are to each other as their base × their alt)

Subst. ~~by ax1~~ $\dfrac{AB}{A'B'}$ for $\dfrac{BD}{B'D'}$:

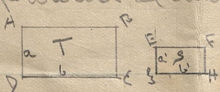
thus: $\dfrac{\triangle ABC}{\triangle A'B'C'} = \dfrac{BD}{B'D'} \times \dfrac{AC}{A'C'} = \dfrac{AB}{A'B'} \times \dfrac{AC}{A'C'} = \dfrac{AB \cdot AC}{A'B' \cdot A'C'}$

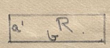
Q E.D.

2 D

III

2 rectangles are to each other as the product of their bases × their alt.





Given rect. T with base b and alt a
" " S " " b' " " a'

Prove $\frac{T}{S} = \frac{a \cdot b}{a' \cdot b'}$

Proof Draw rect R with a' alt of rect S and base b of rect T.

Then $\frac{T}{R} = \frac{a}{a'}$ (2 rectangles having = bases are to each other as their alt. and 2 rectangles having = alt are to each other as their bases.)

$\frac{R}{S} = \frac{b}{b'}$

multiplying the results

$\frac{T}{S} = \frac{a \cdot b}{a' \cdot b'}$ (Ax. 1.)

Q. E. D.

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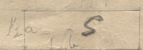
IV

Construct a square = to 1/3 of a given Δ.

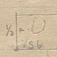
required a square = to $\frac{1}{3}$ T.

$T = \frac{1}{2} a \times b$ (the area of a Δ = 1/2 the product of its base × its alt.)

Take base a line = to base b. and on its extremity erect a line = to 1/2 a.



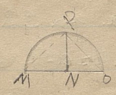
rect $S = \frac{1}{2} a \cdot b$ (the area of a rect. = its base × alt.)



The area of $U = \frac{1}{6} a \cdot b = \frac{\frac{1}{2} a \cdot b}{3} = \frac{1}{3}$ area of T.

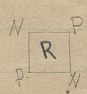
Draw $MN = \frac{1}{3} b$
$NO = \frac{1}{2} a$

with MO as a diameter erect a semicircle.



PN is the mean proportional between MN and NO

for by drawing PM and PO a rt Δ is form. and PN is the alt to the hypt. (rt alt drawn to the hypot. of a rt Δ is the mean proport between the segments of the hyp.)



R is a square for PN is drawn as one side a PN is drawn as another side.

The area of $R = PN \cdot PN$ (the area of a sq. = base × alt)

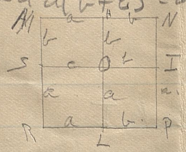
∴ $R = \frac{1}{3} T$

Q. E. D.

Not good
15 — Construction

V

Prove geometrically that $(a+b)^2 = a^2 + 2ab + b^2$ and $a(b+c) = ab + ac$.



$\frac{a}{b}$

Draw RL = a. Draw MR = RP ⊥ to RP.
" LP = b.

Draw MN ∥ RP and NP ∥ MR.

on MR lay off RS = a. Draw SI ∥ RP.

at L Draw LH ∥ MR.

~~at~~ MO, ~~and~~ ON, ~~and~~ RO, ~~and~~ OP are parallellograms and rectangles for (their sides are ∥ and they have each ∠ of the large sq.)

RO is the sq. on a. for SR = RL.

RP = a+b const. MN = a+b const.

RS = a const. ∴ MS = b (the whole is = to the sum of all its parts)

HO = b, NI = b (∥s comprehended between ∥s are =)

OI = b, HN = b (" " " " " ")

∴ ON is the sq on b, for its sides are all = by const. and it is a rect by above proof

OP = ab MO = ab.

∴ MO, ON, RO, OP or sq of $(a+b) = a^2 + ab + ab + b^2) = a^2 + 2ab + b^2$