GEOLOGICAL SURVEY OF KENTUCKY.

N. S. SHALER, DIRECTOR.

ON THE USE

OF

THE TELEMETER

IN

TOPOGRAPHICAL SURVEYS,

BY C. SCHENK.

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ON THE USE OF THE TELEMETER IN TOPOGRAPHICAL SURVEYS.

As I have already said in my topographical report, I made use of the instrument called the telemeter for measuring lengths during the survey I made of a part of Greenup county and of Lawence county. I was principally moved to apply this means by the topographical relations of the country I had to deal with; for there are in this region but few plains over which a direct measuring of lengths, by means of a measuring-staff or chain, is practicable, with anything like the rapidity which was a necessity with me. Owing to the many bends of the roads, it seemed to me pretty difficult and too inaccurate to use an odometer for measures of length and a compass with sights for measuring the angles. Moreover, an odometer cannot be used in the case of rivers, both of whose banks are covered with brushwood, because the apparatus will not work among brushwood. An odometer consists, for the most part, of a wheel which rolls on the ground, and connected with an indicator, and, when pushed by the operator, rolls further. From the number of revolutions, which correspond to a certain amount of road gone over by the instrument, the distance is read off by means of the indicator.

On straight level roads, and especially on railroads, the odometer is a good instrument for measuring lengths, and is more accurate than a chain. But where the roads are winding and bordered by fences, the use of an odometer must entail a great inaccuracy.

The telemeter, used along with an instrument for measuring angles, offered me the following advantage: from the point where the instrument rested I could measure the distances at the same time that I measured the angles, and I could measure the distances through the air above or beneath the many obstacles that were in the way; in the same way I was able to measure very successfully across a surface of water in the case of streams, a thing which cannot be done with chains, staffs, or odometer. These remarkable advantages threw the balance completely on the side of the telemeter.

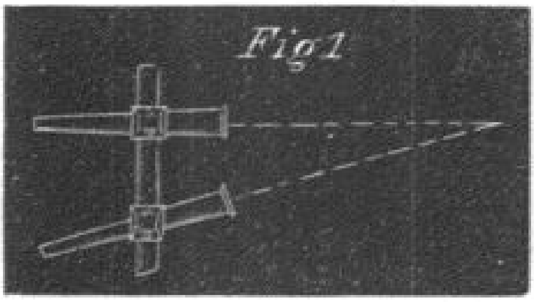
As many looked on with heads shaking disapproval on the method of measurement which I used, and as the telemeter is not sufficiently well known, even by specialists, and in general is not so much used as it deserves to be, I have decided, in accordance with the wish of my chief, to give in the following pages a short explanation of the theory, and its practical application.

THE THEORY.

Theoretically, telemeters are divided, first of all, into two classes. In one class a staff and telescope are used in such a way that the operator looks through the latter towards the former; in the other class of telemeters a staff is altogether dispensed with. We must therefore distinguish between telemeters with and without a staff.

Many telemeters without a staff have been proposed, and even constructed, but so far they have furnished less satisfactory results than those obtained from telemeters with a staff, and are of less value for practical geometry.

Consider the annexed sketch, which is intended to represent two telescopes provided with diaphragms, which can be moved to and fro on a staff in such a way that the angle which their optical axes make with each other remains constant. It is easy to see that the distance of the two telescopes from each other must be exactly proportional to the distance of a point sighted through both telescopes, assuming that the instrument is not moved. This contrivance would be very excellent if only the condition of keeping the angle of sight constant could be fulfilled with sufficient accuracy. This would require a staff of an uncommon grade to move the telescopes on, and one that would not warp; in short, almost mathematical accuracy would be required in the making of the different parts.



One improvement is to use the so-called angle-mirror or prism instead of the telescope and staff; or, as with the case-sextant mirror, one can deduce the distance from the eccentricity of the alhidade and the measured angle.

Angle-mirrors or prisms can be easily put in position, by means of which the engineer, working on a short basis, can determine geometrically a third (inaccessible) point. An instrument of this kind can even be put in one's vest pocket. The relation of the base, which is to serve for measuring the distance of the point which is to be determined, must be decided before getting ready the prisms, since this relation determines the angle B, according to which the prisms must be drawn.

Let D equal the distance from the point where the operator stands to the object; let b equal the base, and *be* $\frac{D}{b} = m$; so is $\cos B = \frac{1}{2m}$. The quotient m may go up to 3, 4, 5, etc.

Military men have already frequently made experiments with telemeters without staffs. One may claim already to measure accurately with prisms, especially if m is not taken too large.

So much for telemeters without a staff. I now pass to telemeters with a staff.

TELEMETERS WITH A STAFF.

These are divided into two kinds. In one kind the length of staff is constant—*i. e.*, the length of staff that can be used for computation (mostly by means of points whose distance from each other is known) is constant; the angle between the two extreme lines of sight is measured, and from this angle, together with the known length of the staff, the distance of the staff from the point where the instrument stands is computed. In the other kind a portion of staff proportional to the distance is used, and the distance is computed from this portion of staff and a constant, which latter depends on the construction of the instrument used.

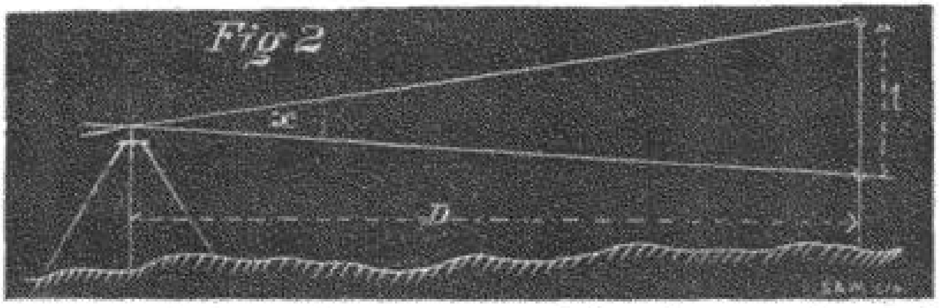
TELEMETERS WITH A CONSTANT LENGTH OF STAFF.

This contrivance consists of a telescope for measuring distances with cross-hairs on one side and a staff of determined length on the other side. After setting the staff and telescope the latter is used to sight the former, and thus the angle, which is wanted in order to bring the cross-hairs from one point of the staff (the blank) to another, is measured. This angle is measured either by moving the telescope, or turning it, or else the horizontal thread itself is moved (Meierstein's telemeter). It is easy to perceive that the accuracy of measurements depends entirely on the accuracy with which the angle is taken, if one leave out of account the correct position of the staff; and in practice this condition is commonly fulfilled by means of finely-cut screws, on which are heads divided so as to determine with accuracy the whole and fractional parts of the necessary rotations, which take place, when the cross-hairs are moved from one target to another.

The telemeters in which, in order to obtain the above result, the telescope is turned through the necessary angle by means of a fine screw, are called Stampfer's telemeters.

As has been already mentioned, the angle which must be known in order to move the optical axis of the telescope during the measuring of the distance from one blank to another, is measured by means of a screw. · Furthermore, it is easy to see, that if u equal the necessary number of rotations of the screw, u is inversely proportional to the distance of the staff from the telescope itself; and hence, from the quantity u the distance itself can be determined.

Let l equal the distance between the two blanks on the staff. Let D equal the distance from the staff to the standpoint of the observer. Then, owing to the smallness of the angle of inclination which the lower line of sight drawn to the target makes with the upper one, it is accurate enough to say $D = \frac{l}{\tan x}$, where x denotes this angle of inclination.



Now the angle is to be expressed by the number of screw-threads that are used in order to bring the telescope through the arc *x*; and from this one writes down $\tan x = c\,u$, where *c* denotes a constant depending of the disposition of the instrument itself, and one obtains $D = \frac{l}{\tan x} = \frac{l}{cu}$.

The constant *c* can be determined as follows: A length *D* is accurately measured on horizontal ground; the instrument is placed at one end of this distance, the staff at the other, and one calculates how great *u* is when *l* and *D* are known, and it will be good to determine *u* several times. Further, we have $\frac{1}{c} = \frac{D}{l}\,u$ = a coefficient which we can call *k*, and then we write $\frac{1}{c} = \frac{D}{l}\,u = k$, hence $D = k\,\frac{l}{u}$.

Stampfer's instruments have the constant $k = 324$, and therefore $D = \frac{l\,324}{u}$. We now see the results of this equation: *l* is constant, and different values assumed for *u* are put together in tables from which one can immediately read off the length *D*, corresponding to the quantity *u*.

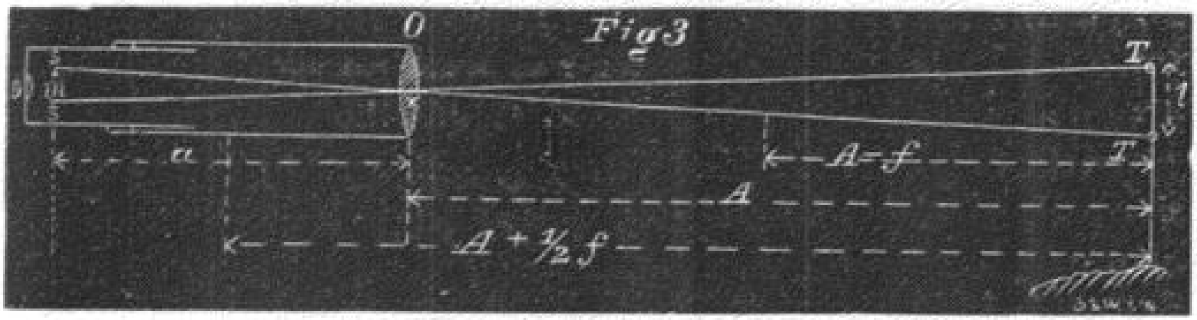
A very great accuracy is claimed for this telemeter by Prof. Stampfer. I have only touched lightly on this telemeter because, unlike the kind I am going to describe, it cannot be adapted to a theodolite which is not specially constructed for the purpose. I now turn to the other kind of telemeters known as Reichenbach's telemeters.

REICHENBACH'S TELEMETER.

This telemeter can easily be adapted to any telescope by drawing two parallel threads across the diaphragm ring of the telescope at any distance from each other.

In order to perceive the way in which this contrivance (the hair micrometer) works in connection with a staff, let us consider, for our purposes, a simple (Kepler's) telescope. Let us assume, further, that the staff is in a vertical position, and that the telescope is directed at right angles to it—*i. e.*, that it is itself horizontal.

Let O denote the object glass. Let o denote the ocular glass. Let m denote the distance between the threads, ss, of the hair micrometer. Let a denote the distance of the position of the image of the staff from the object-glass. Let A denote the distance of the staff itself from the object glass.



Then from the similarity of the two triangles $T\ x\ T$ and $s\ x\ s$, we have the following simple relation: $\frac{a}{A} = \frac{m}{l}$ . . (I).

Further, according to a formula in dioptrics, $\frac{1}{f} = \frac{1}{a} + \frac{1}{A}$, whereby f is meant the focal distance of the object lens. This formula will be found developed in any good text-book on physics. Taking now this latter equation with respect to a, we get $a = \frac{f}{1 - \frac{f}{A}}$. Substituting the value of a in the first equation, we have finally, $A - f = \frac{f}{m} l$ . . . . (II).

That is to say, the distance from the anterior focus of the object-glass to the staff is proportional to the length of the portion of the staff cut off on the staff by the threads.

Reichenbach's telemeter is constructed with this proportion as a basis.

The proportion between the focal distance f of an object-glass and the distance m between the spider-webs, which latter are drawn tightly across the diaphragm ring in the telescope is a constant quantity, viz: $\frac{f}{m} = i$. Furthermore, in the same telescope the distance between the axis of revolution and the object-glass is also constant. But the middle point of the axis of revolution is also the middle point of the instrument from which one wishes to measure the distance to the staff. If now, for our further investigations, we adopt a telescope whose object-glass stands $\frac{1}{2}f$ from the axis of revolution of the tube, which is approximately right, then, first, the distance of the staff from the middle point of the instrument is expressed by $A + \frac{1}{2}f$.

If $1\frac{1}{2}f$ is added to both sides of equation (II), and if for $\frac{f}{m}$ its value i is written, we have $A + \frac{1}{2}f = i\,l + 1.5\,f$ (III).

If, then, in a telescope the constant quantities i and f are known, then by means of (III) for every distance of the staff from the middle of the instrument—that is, for every value of $A + \frac{1}{2}f$—a quantity which we will hereafter denote by E—the corresponding portion of the staff can be calculated and the staff divided accordingly.

In the case of Ramsden's eye-piece, which is here most commonly used, and in that of Kepler's telescope, or by the excellent Kellner ocular, which I now use, this formula holds true without exception. In the case of Hughen's telescope, however, the intervention of the collecting lens causes the rays coming from the objective glass to be drawn together; and in this case, to complete our investigation, further considerations are necessary. Since, however, Hughen's eye-piece is but little used in this country, I will not stop to investigate it here; still, I will add, that, for my own part, I have hitherto used a Hughen's eye-piece, and that I shall be glad to help any one who desires explanations about it.

The eye-piece, mostly used with measuring-telescopes here in America, is, so far as my experience extends, the so-called terrestrial or Rheitas eye-piece, which may be considered as arising from a combination of Ramsden's and Hughen's. This eye-piece is like the simple one, or like Ramsden's, in that there is no collecting lens between the cross-hairs and the object-glass.

When one wishes to make a division of the staff according to the formula given above, it should be observed that if the threads are placed much apart, a long staff is needed, while if the threads are placed close to each other, a small staff suffices; small staffs, however, give less accurate results than long staffs, and many persons hold that one should not take $\frac{f}{m}$ larger than 70.

This value is not very convenient, especially because it requires a long staff that must be divided in a special way. If 100 is used as unit of division, one has the advantage of being able to use every leveling staff for measuring distances, although, indeed, the accuracy of measurement is somewhat less.

I use 100 as the unit of division, and, therefore, for a distance of 1000 feet, I need a staff about 10 feet long.

Let us now investigate the case where we wish to use for measuring distances a leveling staff that is already divided, and let us see what the practical results would be.

We have given the leveling staff with decimal division, therefore we must have $i = 100$.

We wish further to measure the focal distance of the objective glass; this can be done with sufficient accuracy by means of compasses; after a distant object—a star—has been sighted to that there is no parallax, one has only to measure the distance between the object-glass and the diaphragm.

We have now $i = 100$, and we have also determined f. We can, therefore, from equation (III) or equation (II) or from $\frac{f}{m} = 100$—calculate the distance between the threads, and then place the threads in position. It is necessary to calculate the distance between the threads as nearly as possible, in order that the threads may be placed in their proper position as nearly as possible, so that it may not be impossible to effect the small correction which is mostly required later, and which should be rendered possible by means of some contrivance such as a screw. For, with ordinary means, it will be impossible to place the threads with the accuracy which does not show itself till under the magnifying power of the eyepiece.

When we have placed the threads in position upon the diaphragm ring, and the latter in the telescope, we can go on to investigate what the telemeter accomplishes and with how much accuracy it works.

First of all, we have equation (III), viz: $E = il + 1.5 f$ to take into account, and to determine the portion which must be used for certain distances E.

If, for example, we make E successively equal to 100, 200, 300, etc., the above equation solved with respect to l gives for

$$E = 100, \quad l = \frac{100 - 1.5 f}{i} = 1 - \frac{1.5 f}{100}$$

$$\text{for } E = 200, \quad l = \frac{200 - 1.5 f}{i} = 2 - \frac{1.5 f}{100}$$

$$\text{for } E = 300, \quad l = \frac{300 - 1.5 f}{i} = 3 - \frac{1.5 f}{100}$$

that is, *when the staff is 100 feet off the portion of staff used is 1 foot* $- \frac{1.5 f}{100}$ *in length;* when the staff is distant 200 feet from the centre of the instrument *the length of staff used is* $2 - \frac{1.5 f}{100}$ *feet, and so on; so that a portion of staff less than 1 foot corresponds to the first 100 feet, and exactly one foot more of staff is required for every additional 100 feet.*

The point on the staff which is determined by the quantity $\frac{1.5 f}{100}$ has been fitly called the zero point. In this connection I should mention, that for this determination I supposed a telescope, whose pivoting axis was ½f from the object-glass; if this quantity were different, it would have to be introduced with its proper value. For example, in the case of a telescope, the focal distance of whose object-glass was six, and between the axis and object-glass three, decimal inches, the portion of staff corresponding to a distance of one hundred feet would be 0.991, and for two hundred feet, 1.991, etc.

When this zero point has been so computed it is marked on the staff itself, and, in measuring, the upper thread is always directed to this zero point, while, by means of the lower thread, the distance is read off on the staff. If one has to work with a staff without zero point, it is necessary to add to the distance taken between any numbers the constant 1.5 *f* in order to get the correct distance. Accordingly, as I have here pointed out, it is entirely wrong to place one's threads in such a way that, for a distance of one hundred feet, they cover just one foot of the staff. This arrangement is assumed by many to be correct, as I found to my grief; it is even set forth as correct in instructions about the use of instruments. One has only to use an instrument with a considerable focal distance in order to perceive, that if the threads cover one foot of staff for a distance of one hundred feet, the measurement of great distances becomes very inaccurate. With my telescope I would have made an error of fifteen feet in 1,000′ distance if its threads had been set in the faulty way I have mentioned.

When one's telemeter has been put in order, and the staff also is in order, the zero point having been determined, the next thing is to measure on an even plane, as accurately as possible, a suitable extent of ground from five hundred—one thousand feet with a staff or chain. A straight railroad rail is peculiarly suitable for a good measurement, which can be accurately taken by means of a steel tape measure. Next the instrument should be placed at one end of the measured strip, the distance staff should be set vertically up at the other end, and the engineer should examine whether the threads have the separation which corresponds to this distance, and whether they cover exactly the zero point above, and—for example, for a distance of one thousand feet—the ten-foot partition counted from above.

A variation from the present disposition of the instrument can be obtained, if necessary, by pushing the slits on which the threads rest, which movement is practically effected by adjusting screws.

It will be good to try the telemeter on many lengths that have been accurately measured, and it is specially advisable to measure small distances with it. If all comes out right, one can trust with safety to his telemeter.

The above remarks were made under the supposition that the staff was placed vertically and the telescope horizontally. In practice such a use is seldom made of the instruments, except in leveling; on the contrary, the sights are inclined either upwards or downwards. We have still to investigate in what way the results of measurement are modified by this departure from the previous hypothesis.

We have further to consider whether the staff shall be placed vertically; or perpendicularly to the line of sight of the telescope.

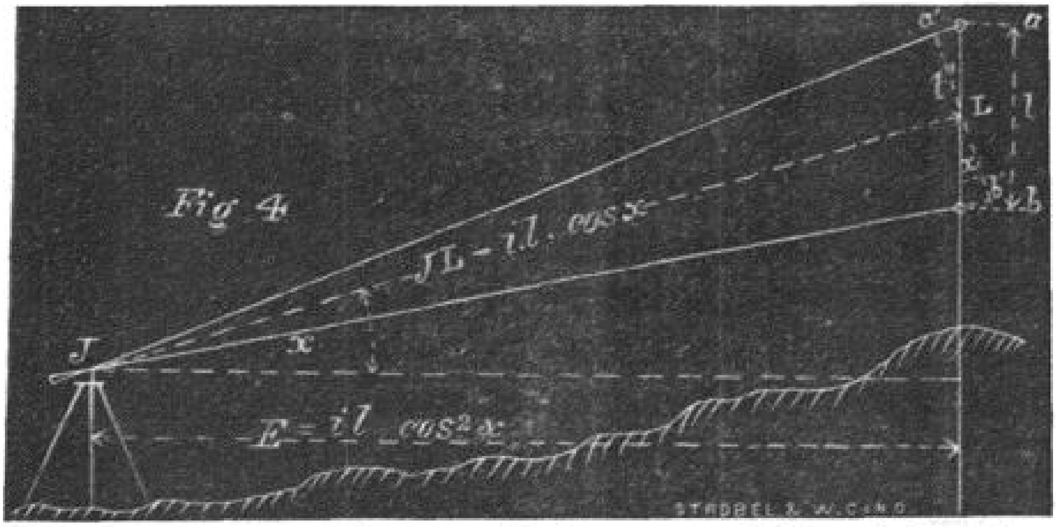
The staff can be placed vertically by hand, by a level, by a plumb-line, or by balancing the staff on a point placed under it. I make use both of balancing and plumbing, according to the nature of the work.

The staff can be made perpendicular to the line of sight by placing a *diopter* on the staff, and then sighting the instrument from the staff.

REDUCTION OF OBLIQUE LENGTHS.

If one sights a vertical staff under an inclined angle, then, on account of the oblique sighting, a larger part of the staff will come between the threads than corresponds to the direct distance. The length of the portion of staff so sighted can be read off directly. The angle x, under which the staff is sighted, can also be read off. Therefore, we have the data for reducing.

The portion of staff $a' b'$ corresponding to the distance $J L$ is the correct one, while as a matter of fact the greater portion $a b$ is read off, and we wish, therefore to deduce $a' b'$ from $a b$.

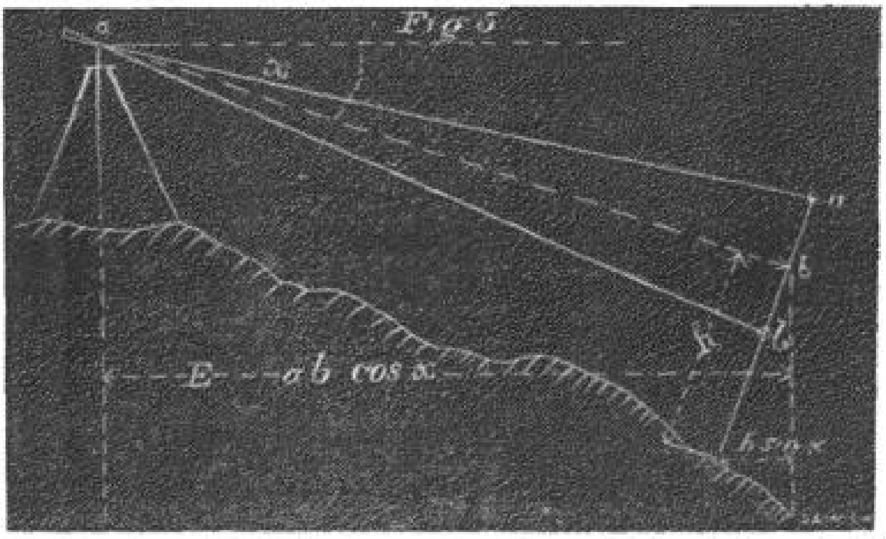


Owing to the smallness of the arc which a' b' **subtends,** a' b' **can be expressed with sufficient accuracy by** $l' = l \cos x$, **when** $ab = l$ **and** a' b' $= l'$ **the value** l' **so obtained corresponds to the oblique distance** J L, **which is denoted by the expression, oblique length. This quantity,** J L, **however, must still be brought down level with the horizon; it cannot yet be called the correct horizontal measure. This quantity** $l \cos x$ **must itself be reduced, which is done by multiplying again by** $\cos x$, **so that we have** $E = il \cos^2 x$.

The quantity $\cos^2 x$ **can be very easily deduced from** x **itself. Thus, if it is wished to carry out the multiplication in a graphic way (which is a very convenient operation if one wishes to put the lengths on a scale, in order to introduce them into a map), one has merely to calculate the values of** $\cos^2 x$, **to consider them as** *cos* **of an angle, and to set these angles down in a diagram, by means of which the whole reduction can be completed with compasses. (Jordan's diagram.)**

If the staff is placed at right angles to the line of sight, the necessary reduction to horizontal values can be performed by the help of the following considerations.

If a b **again stands for the portion of the staff covered by the threads, then in both figures the horizontal correction will simply be:** $a\ b \cos x = E$. **But it will be perceived that, owing to the oblique position of the staff, it rests at another place than that which** E **requires. This reduction depends also**



on whether x is an angle above or below the horizon, and on how high on the staff the middle line of sight reaches.

If we call this length, which changes for every distance, h, then, besides the above reduction, $h \sin x$ will have to be added to E for an angle of elevation, and subtracted from E for an angle of depression, in order to determine the point where the staff touches the ground.

The data of reduction for a given staff, and for different values of E and x, have been tabulated, and by means of these tables the reduction can easily be effected.

With respect to the telescopes that are to be used for measuring with telemeters, they must be of the best quality, and must possess great clearness, and especially definition, together with great magnifying power.

ON THE ACCURACY OF LENGTH MEASUREMENTS.

Experience shows that in all measurements a deviation occurs from the real length. I do not wish to include among errors so committed those which arise from inaccurate observation and inaccurate reading; such mistakes can be avoided; I refer to mistakes arising from the imperfection of our tools and senses. An idea of this kind of error may be got by measuring several times a length of 100′ with a 1-foot measure, and finding that each result deviates slightly from the preceding one.

It is possible, by repeatedly measuring the same quantity, to get an approximation of its absolute value. It is advisable to measure the same length twice over, if for no other reason, because a tolerable agreement of both measurements gives the certainty that no grave error has been committed.

By the method of the smallest square one can find the mean error out of several measurements of a quantity, or the mean error of one single measurement. This method is of use to the practitioner, because he can make clear to himself the errors which he has to fear during his work. It is especially used where, after taking great care with the measurements, one wishes to bring the final result still nearer the probable true value.

In the following tables are arranged, as far as I know them, the results which have been obtained by the method of the smallest square, from the most accurate and from less accurate measurements:

MEASUREMENT OF BASES.

Mean errors in a single measurement of the length of one kilometer = 1,000 meters = 3,280 English feet.

Year.		Millimeters.	English inches.
1736.	Base of Yarouqui, in Peru, two measurements with wooden staves, from 15 to 20 feet long.	16.4	five eighths.
1736.	Base of Tornea, in Lapland, two measurements with wooden staves.	20.2	eleven sixteenths.
1739.	Re-measuring of the Picard base under Juvisy by Cassini	63.2	two and a half.
1819.	Schwerd's small Speyer base, two measurements, 859.4409 meters long.	1.5	one sixteenth.
1834.	Base line of the measurement of a degree in East Prussia	2.2	three thirty-seconds.
1846.	Base line near Berlin for the coast survey, two measurements	1.6	one sixteenth.
1858.	Spanish base of Madridejos, twice measured	0.4	one forty-eighth.
1860.	Small Spanish base of Ivice, measured four times	0.3	
1868.	Austrian base in Dalmatia, two measurements	0.7	

MEASUREMENT WITH STAVES.

The mean error of one measurement derived from measuring twice, the lengths measured being different.

Length in feet.	Mean errors of a measurement in decimal inches.
300	1.098
600	1.553
1,000	2.005

CHAIN MEASUREMENTS.

This is derived from over 500 measurements taken twice of lengths going up to 1,000 feet, with chains of from 30 to 50 feet long:

MEAN ERROR OF ONE MEASUREMENT IN DECIMAL INCHES.

Length in feet.	On sandy ground.	On loamy ground.
300	5	3
600	7	4
1,000	10	6

Whence it is evident that measurement with a staff is three times as accurate as measurement with a chain.

In ordinary measurements, which are made with somewhat less care, the mean error proves to be as follows: on hard ground, 1 ÷ 1000; on washy or soft ground, 1 ÷ 500; on common ground, 1 ÷ 700.

When a chain is used, its tension should carefully be observed in order to compensate it. The depression of the chain, owing to careless stretching, produces an error which increases with the square of the depression.

OF THE MEASUREMENT OF DISTANCES BY MEANS OF REICHENBACH'S TELEMETER.

I obtained the following results on a railroad track on which the distances had been accurately measured with a steel ribbon; the distance was read off three times for each place where the staff was put up:

WITH THE TELEMETER—MEASUREMENT.

Length measured with the Steel Ribbon.	First.	Second.	Third.
100	100	100	100
200	200	200	200
400	399.5	400.5	400
600	599.5	600	599
800	799	799.5	800
1,000	999	1,000	1,000
1,200	1,199	1,201	1,198

Whence the deviation from the accurately measured length is at the utmost $1 \div 600$. These measurements were made with great care, in quiet weather, and with good light. Such accuracy is surely not to be obtained by a day's work. According to the results obtained by other observers, the accuracy amounts to somewhat less; the error is given as from $1 \div 400$ up to $1 \div 300$. But, taking merely the accuracy obtained with the last error, and the telemeter is still a very good instrument for topographic work.

Moreover, it is easy with a telemeter to take several readings instead of one, and so to increase the accuracy; so much so that four readings double the accuracy.

It is of special importance in the measuring of distances that the staff, if it is used in a perpendicular position, should be satisfactorily held perpendicular; and this is pre-eminently true on sloping ground, since the very considerable errors are made by carelessness in placing the staff. If, for example, a ten-foot staff is held so that its deviation from the perpendicular amounts to one foot, and if the staff is sighted at under an angle of five degrees, this would already bring about an error of one per cent. in the length.

With a telemeter we have also to consider the state of the air with respect to rest or motion, the light, and the condition of equilibrium of the lower layers of air. In summer, during the hot part of the day, the sunlight brings about such a trembling of the images of the sighted object that sighting and reading off become very difficult operations. When the sky is clouded the objects are quietest, and one can then work very comfortably.

The advantages of the telemeter are specially manifested in a favorable light in making topography which must be quickly completed, because the measurement of distances takes place from the same stand of instruments from which other objects, such as houses, etc., are placed in; as unevennesses, bushes, etc., are disregarded, as long as one can see through and over them. If one has to survey a stream, and is unable to see along on the shore, owing to weeds, trees, or plantations, one has only to go to the water, and if one can only get a place to stand upon, there is nothing to prevent the measurement along or across the water.

The rapidity with which the work progresses depends naturally on the ground and material which one has to deal with. From six to eight miles is a good day's work. Sometimes I have measured ten miles in a day.