REPLACEMENT OF DAIRY COWS
A Multistage Decision-Making Problem

By

John C. Redman and Lily P. H. Kuo

•

RESEARCH REPORT 1 : December 1969

University of Kentucky :: Agricultural Experiment Station
Department of Agricultural Economics
Lexington

CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

REPLACEMENT OF DAIRY COWS

A Multistage Decision-Making Problem

by

John C. Redman and Lily P. H. Kuo

THE PROBLEM

In a dairy enterprise, considerable quantities of time and other inputs are allocated by the operator to raising or acquiring replacements for the producing cows. The crux of the dairy cow replacement problem is basically concerned with the optimum time at which a cow is to be replaced. Milk production as a biological process increases to a maximum point and then decreases, both within one lactation and between lactations within a cow's life span. The problem has two dimensions. First, a dairy producer must decide whether to replace a cow during a given lactation, and second, if the decision is to replace her, when to do it. However, in this study, the attention was concentrated on the problem of determining the best lactation number for replacement.

The replacement of dairy cows is a multistage decision-making problem since the best decision depends upon subsequent and preceding decisions. The total planning problem for multiperiod production of the firm consists in determining the value of all input and output variables for all time periods within a certain planning span of time. The replacement problem is concerned only with determining value of durable input variables and only with cases where one durable asset is replacing another already in operation, instead of simply being added to the total number of such assets. For example, if a farmer decides to add one dairy cow to those already in production at a given time, this is not a replacement situation but one of increasing the size of the dairy operation.

Further, it has been argued that a replacement situation exists only when the level of production is not changed after the action of replacement. It would be extremely difficult to maintain the distinction between replacement and scale adjustment in an operational model. For instance, the farmer replaced an old cow of low productivity with a young one with higher productivity. By feeding less to the young cow, it might be possible to maintain milk production at the same level as before; however, the economic result from such a policy would not be a proper basis for studying the economic effect of the replacement. For the purpose of this study, the term "replacement" will refer to any case where one durable capital item is substituted for another one, whether the replacement is accompanied by a change in output or not.

PREVIOUS STUDIES

Very few economic studies have dealt explicitly with dairy cow replacement. So far there are only two studies known by the authors. One, published by Jenkins and Halter [14], must be considered a methodological study in the field of dynamic programming. It uses the dairy cow replacement problem as an illustration of given principles.

The second study was accomplished by Giaever [11], using the method of dynamic programming with Markov processes as developed by Howard [12].

Several research projects in the field of dairy science have dealt with current replacement patterns and disposal causes. However, there is no known literature in this field which attempts to determine the optimal replacement pattern for dairy herds in terms of maximizing profit.

EMPIRICAL MODEL FOR DAIRY COW REPLACEMENT

Objectives and Hypotheses

The principal objectives of this study were:

a. To test the hypothesis that by adopting the optimal replacement policy of dairy cows, the dairy farm can increase net returns over the life span of the enterprise.

b. To show how an optimal replacement policy can be obtained by the dynamic programming model.

c. To determine the economic loss to dairy producers who follow replacement policies which deviate from the optimal.

d. To study the sensitivity of optimal replacement policies to variations in prices and in other parameters in the model.

e. To gain experience with practical problems which have to be faced when applying this kind of models.

Assumptions Necessary for This Study

a. In any enterprise period, it was taken as certainty that replacements in every specified lactation (i. e., from 1st to 7th lactation) were available.

b. The deviations of random replacements of cows on account of diseases, injury or death, were normally distributed. The probability of random replacement was constant over years.

c. The deviations of production and feed consumption among cows were normally distributed.

d. The motive of the dairy producer was profit maximization over the life span of the enterprise.

e. The dairy producer followed a given "feeding system" which may have varied over time as a result of price variations, but which, at any given time, was the same for all cows in the herd.

f. All cows which had not been replaced before were replaced at the end of the seventh lactation. In actual practice few cows are kept longer than to the end of the sixth lactation.

g. There was no significant difference between replacement with a purchased animal and replacement with a self-raised heifer.

THE METHODOLOGY

Background of Theoretical Development

As pointed out by Preinreich [15], the general theory of replacement is simply the theory of maxima and minima. A solution to the optimization problem is attributed to the German forester Faustman [9] as early as 1849. Preinreich was the first to apply contemporary methods to the problem. However, he discussed only the problem of replacing industrial equipment. Gaffney [10] has dealt with the optimal harvest age of timber. Faris [8] discussed, more generally, replacement patterns for biological production involving time. Winder and Trant [22] have discussed some of Faris' replacement rules and provided a more rigorous analysis of the principles.

Bellman [2] described the general principle of dynamic programming which provides a convenient analytical and computational framework for dealing with the replacement problem.

Delineation of the Method Used

The replacement problems faced by dairy farmers are multistage decision problems, i.e., time factor plays an important role in the replacement problems. Since the entire multistage decision process is considered in most conventional farm management models as essentially one stage, it is difficult to find the effective analytical solution of a problem with many dimensions, i.e., a large number of equations to be solved simultaneously. Dynamic programming is mainly designed for the multistage decision problems. The advantage of this method is that it reduces one problem of many dimensions to a problem of one dimension. This makes the problem analytically more tractable and computationally vastly simpler.

Some additional terminologies need to be introduced, and a brief discussion further of what is meant by a multistage decision process is presented below.

A *stage* of the process is associated with an interval of time, and the time interval is assumed constant from stage to stage.

The *state* of a system at any time is specified by a set of variables. In the course of time, this system is subject to changes of the variables describing the system undergoing transformation.

A process may be assumed in which a choice of the transformations is available to be applied to the system at any time. A process of this type is called a *decision process*, with a decision being equivalent to a transformation. If a single decision is made the process is called a *single-stage process*; if a sequence of decisions are to be made, then the term *multistage decision process* is used.

In a replacement decision process, a physical (or biological) system prevails whose state at any time is determined by the values of a set of physical and economic variables. At certain times, decisions must be made which affect the state of the system and which are based on the prevailing state of the physical and economic variables. The outcomes of past decisions are used to guide the choice of future ones. This sequence of decisions is called a *policy*. The purpose of the replacement decision process is to maximize some function of the variables describing the final state. A policy which is best according to some preassigned criterion will be called an *optimal policy*.

In livestock enterprises, recurring decisions concerning the replacement policy in a biological system are faced. If profit maximization is the goal, the decisions are based on the expected net returns of the present animal on hand compared with the expected net returns from the replacements, the cost of replacing the present animal, the expected revenue from the sale of the present animal, and the expected purchase price of a replacement. The purpose of the decisions is to maximize the profit over the life span of the enterprise.

An example of a process from a physical system is the determination of a replacement policy for farm equipment. In this case, it is necessary to consider the output of both old and new equipment, the maintenance cost for the old machine compared with that required by a new one, the purchase price of the new machine, and the trade-in value of the old machine.

All members of the family of replacement models rest on the same set of assumptions. (1) A "chain" of replacement is present, where one asset is always replaced by another of identical or similar type. (2) Variation in intensity of use of the asset is not considered. Thus, these models do not consider the case where the life-span of one asset depends on how heavily it is being used. (3) Income per time unit, outlays per time unit, and salvage value of the old asset all are known mathematical functions of age of the asset. (4) Capital is available at a given rate of interest. (5) The objective criterion is maximization of the present value of future net returns.

The replacement decision process for the continually operating dairy enterprise used in this study can be represented by a recursive equation. The approach by which the equation arises is to consider the set of all possible sequences of decisions, i.e., the set of all feasible policies, compute the returns from each such policy, and then maximize the return over the set of all policies. An optimal policy has the property that whatever is the initial state and initial decision, the remaining decisions must constitute an optimal policy with regard to the future state of the system.[1] Thus, the approach can be expressed by a recursive equation as,

$$\Pi_{i,t} = V_{i,t} + \text{Max } (P_j \cdot NS_{j,t} + Q_j \cdot NF_{j,t} - V_{j,t} - S_{j,t} + \lambda \cdot \Pi_{j+1,t+1})$$

This equation determines the decisions that will maximize net returns within a certain planning time span, T, when the initial condition, $\Pi_{j+1,0}$, are specified. The sequence of decisions shows the most profitable age at which cows are to be purchased in the beginning of the enterprise and subsequently, the most profitable replacements will be made in each enterprise period. In each enterprise period, the farmer can either keep the cow for another time period or replace her with a cow of a different age.

A description of the variables and a discussion of the relationships among the variables may be made as,

t - planning time span, enterprise period (= 1,2,3, . . ., T)
i - lactation of cows on hand (= 1,2,3, . . ., I)
j - lactation of replacement cows (= 1,2,3, . . ., J)

The term, $\Pi_{i,t}$, is the dependent variable for which the recursive function will be solved and is the maximum net return, discounted to present value, for time period t and subsequent time periods from a cow in lactation i.

The market value of the present animal, $V_{i,t}$, and the market value of replacements, $V_{j,t}$, are based on the value of beef plus the expected net returns of dairy production from present and subsequent lactations. Each is a function of current production levels, the number of lactations remaining for the animal, the price of milk, beef and feed, and the supply and demand for dairy cows. It is assumed that $V_{i,t} = V_{j,t}$ when i = j. However, the difference, if any, between $V_{i,t}$ and $V_{j,t}$ when i = j in the real market will be included in the transaction cost ($S_{j,t}$).

The term, $P_j \cdot NS_{j,t} + Q_j \cdot NF_{j,t}$, expresses the expected net returns from a replacement cow in lactation j in time period t. There are four variables involved in determining the expected net returns. The probability of success of lactation j, P_j, and the probability of failure of lactation j, Q_j, are stochastic factors in the equation. Failure of a dairy cow is defined as the removal of the animal from the herd for sickness, injury or death. The variable $NS_{j,t}$ is the net return from a cow of lactation j in time t if she completes lactation j, i.e., the net returns from a successful lactation. The net returns from failure, $NF_{j,t}$, is the salvage value of a cow of lactation j, in time period t if she fails to complete lactation j.

The transaction cost, $S_{j,t}$, includes the commission charge, cost of transportation of the cows to and from the market, labor cost and any difference

[1]This is the basic principle of optimality [2, p. 83].

between the selling price of the present animal and buying price of replacements when they are in the same lactation. There will be no transaction cost if the cow is kept another time period, i.e., $S_{j,t} = 0$ when $i = j$.

The discount factor, λ, is determined by interest rate. By using the discount factor, outlays or revenues incurred during different time periods can be made comparable by discounting them to the same time period, the present time.

The term, $\Pi_{j+1,\ t+1}$, is the maximum net return, discounted to present value, of the decision process for a cow of age j + 1 in time period t + 1. This quantity represents the net returns to the enterprise if the optimal policy is followed in future time periods. The subscript, j + 1, is used instead of the j because in the next time period, t + 1, a cow must be one year older.

The meaning of the equation and the relationships between the variables can be expressed with less symbolic notations.

At any time, t, the expected maximum net returns, discounted to present value, for any cow i is measured by:

The market value of the cow, which includes the meat value of the cow and her potential milk production for (t)th and the subsequent time period.

plus

The maximum net returns that can be obtained by replacing the present animal with cows of any of j different ages.

The maximum net returns of each replacement cow are measured by:

The expected net returns from the replacement in time period t, which includes expected net returns from milk production in time period t and meat value of the animal.

minus

The purchase price of the replacement cow, which includes her meat value and potential milk production.

minus

The cost of transaction.

plus

The discounted net returns obtained in the future time periods when the optimal replacement policy is followed.

The equation was designed to represent a recurring replacement decision process of a dairy production enterprise. The length of time between decisions may be of any duration. However, given time duration, the other variables in the equation will be specified by definite characteristics.

Since one of the most distinct features of the recursive model is that the analysis of each time period is dependent on earlier time periods [5] the initial state of the system, $\Pi_{j+1,0}$, should be specified for the purpose of solving the equation. The procedure is to begin with the last enterprise period and proceed backward through time to the beginning of the enterprise. Because the only policy open at the end of the enterprise is to sell the cows regardless of age, the selling price of the cows represents the maximum returns of the replacement policy, i.e., the selling price is used initially as the value for $\Pi_{j+1,0}$ when deriving the replacement policy for the enterprise period next to the last. Without the possibility of carrying out this procedure, there would be no means of giving a value to the $\Pi_{j+1,\ t+1}$ term which represents the net returns of the optimum policy in future enterprise periods.

Another characteristic which should be mentioned here is that a single (ixt)-dimensional problem has been reduced to a sequence of ixt one-dimensional problems. The utilization of structural properties of the equation and the reduction in dimension combine to furnish computing techniques which greatly reduce the time to solve the problem.

As discussed before, the equation will be solved by using the state of the system at the end of the enterprise as the initial value of $\Pi_{j+1,0}$ and proceeding backward through time to a period t which equals some preassigned value T. The enterprise periods are relabeled from the specified initial position, i.e., instead of indexing the enterprise periods as t = 1,2,3,···,T, they are now indexed as t = T,T-1, T-2,···,3,2,1. The recursive equation can now be written as,

$$\Pi_{i,t} = V_{i,t} + \text{Max}\ (P_j \cdot NS_{j,t} + Q_j \cdot NF_{j,t} - V_{j,t} - S_{j,t} + \lambda \cdot \Pi_{j+1,t-1})$$

Once the value $\Pi_{j+1,0}$ is specified, the procedure is to add to it the expected net returns of production from the possible replacements of j different lactations and subtract the transaction cost when $i \neq j$. Having then a vector of j possible returns from replacements, the maximum is selected and added to the market value of the cows on hand of lactation i in time period one to obtain $\Pi_{i,t}$. This $\Pi_{i,1}$ value is used for the $\Pi_{j+1,1}$ value $\Pi_{j+1,t-1}$ in calculating the returns from the j possible replacements in enterprise period two; the maximum of these returns is added to the market value of the cow on hand of age i, resulting in $\Pi_{i,2}$. Similarly, the $\Pi_{i,2}$ value is used for the $\Pi_{j+1,2}$ value in determining the value $\Pi_{i,3}$. The same type of iteration is made for each enterprise period until t = T.

The solution of the equation was easily carried out with the use of an electronic computer because the recursive equation is solved by successive iterations, i.e., at each time t, the same procedure, computational-wise, is repeated. This study was programmed in Fortran IV for the IBM 360 computer which made the problem with its large number of dimensions, i.e., processes involving a large number of possible replacements and a large number of planning periods, relatively easy.

Variables Involved and Their Determination

Three hundred and fifty cows were selected at random from the Holstein D.H.I.A. farms in Kentucky. Three hundred and ten observations were used, while the remaining 40 cows were excluded from the sample because of incomplete information.

The 1965 milk production of the sample cows was collected from the D.H.I.A. Lactation Report of the U.S.D.A. and University of Kentucky Cooperative Extension Service.

To reduce variation of production owing to different quality of cows, the production records were separated into three groups according to the production level. Since total production of a cow varies with calving interval and age, it is necessary to standardize the production record in order to decide to which production level the cow should belong. The 305 days' yield at age 6 was used to measure the productivity of a given cow. Three production levels were defined: (a) the standardized production of less than 12,000 pounds, (b) that between 12,000 and 15,000 pounds, and (c) that above 15,000 pounds. One quadratic equation was fitted to each production level as follows:

$$\bar{x}_1 = 5803.42 + 2162.37L - 218.11L^2$$
$$\bar{x}_2 = 9142.37 + 1782.90L - 166.01L^2$$
$$\bar{x}_3 = 11241.04 + 2184.99L - 180.28L^2$$

Where $\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$ are estimated productions at three different levels, (Appendix Table 2) L is the lactation number. These equations are plotted in Fig. 1.

Using the formula $d^2 = \frac{z^2 s^2}{n} \frac{N - n}{N}$, the precision of estimates from these three equations were calculated [24]. Where $d= |\bar{X}-\bar{x}|$, is the precision of the estimate, $\bar{x}$ is the estimated value, $\bar{X}$ is the true parameter in the population, z is the reliability coefficient, s^2 is the variance, N is the size of population, n is the sample size. With reliability 95 percent, the precision of the estimates is within 205 pounds, 153 pounds, 376 pounds for $\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$, respectively. The estimated milk production by lactations was used for the parameter of milk production in 1965. The production of other years was obtained by multiplying the index of production by these estimates. The index of production was calculated from the Dairy Herd Improvement Records (Appendix Table 3 and Fig. 2).

Data on body weight of the sample cows were obtained from Kentucky Dairy Herd Improvement Association Records Monthly Report. One arithmetic mean was found for each lactation. Using average weight of first lactation as the base, the index of body weight by lactation was calculated (Appendix Table 4).

Information on feed quantities actually given to individual cows at given lactation was not available. Data on average yearly feed consumption for the Holstein D.H.I.A. herds in Kentucky for years 1956-65 were obtained from the Kentucky D.H.I.A. Yearly Herd Summary [16]. Feed consumption for cows at different lactations was determined by modifying average herd consumption by the index of body weight. The index of average herd consumption in years 1956-65 is shown in Figure 2 and Appendix Table 7. Number of cows failing and total number of cows by lactation and production level were obtained by Jenkins and Halter from IBM cards of Pennsylvania D.H.I.A. program in 1960.

Data on market value of dairy cows by lactation were not available. Market value of an animal for purposes of this study was estimated using data obtained from Blue Grass Stock Yards and Clay-Wachs Stock Yards in Lexington, Ky. The market value of other years was adjusted by price index of dairy cows for each year. The transaction cost involved in marketing was estimated from data obtained from the same source.

Prices of milk, feed, beef and dairy cows were obtained from *Agricultural Statistics*, U.S.D.A. Bureau of Agricultural Economics, Division of Agricultural Price Statistics. The index of prices was calculated and is shown in Appendix Table 6 and Fig. 3.

As mentioned before, the discount factor is determined by the interest rate. In this study, a decision-maker willing to accept a 6 percent interest rate was assumed. Then, for the purpose of examining the sensitivity of interest rate to the replacement model, interest rates of 8, 10, 15, and 20 percent were used. In all cases the interest rate was assumed constant over years.

In the previous sections, the length of time between decisions has been discussed. With different time duration, other variables in the equation will be specified by different values. In this study, the decision interval was assumed to be equal to the enterprise period. An enterprise period is defined as the time from the beginning of a lactation to the beginning of the next, and corresponds to one year. The life span of the enterprise is given as 10 years, 1956-65. As mentioned before, this is a backward approach, the starting point for solving the equation being the end of the enterprise. The enterprise periods are indexed as:

$$t = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$$

End of Enterprise Period (1965) ↓ t = 1	Beginning of Enterprise Period (1956) ↓ t = 10

It is assumed that all cows which have not been replaced before will be replaced at the end of the seventh lactation. The subscripts i, j appear as: $i=1,2,3, \cdot \cdot \cdot, 7$; $j=1,2,3, \cdot \cdot \cdot, 7$. With time interval given as one year, the other variables are specified.

The term on the left hand side of the equation, $\Pi_{i,t}$ is the dependent variable for which the equation will be solved.

The variables on the right hand side are all independent variables and will

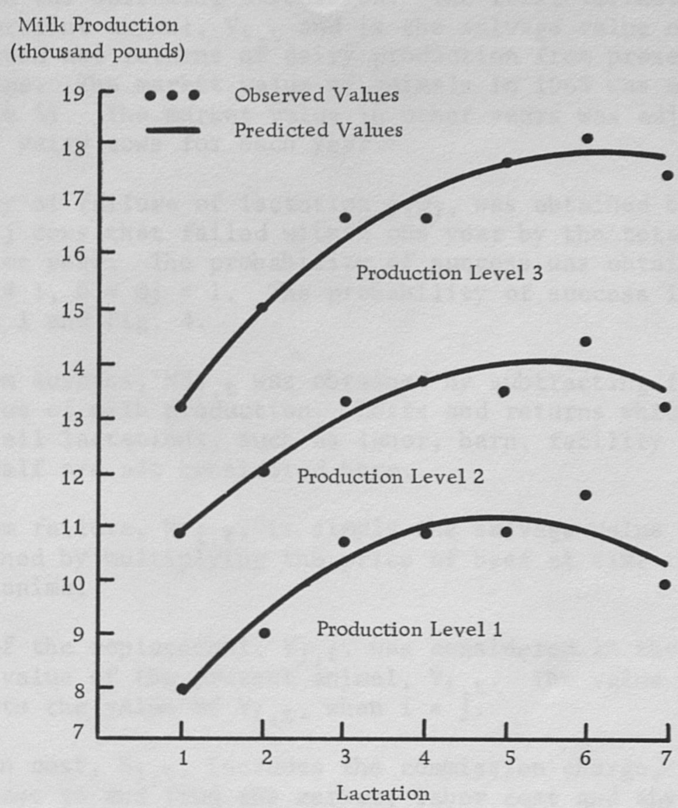

Fig. 1.—Milk Production by Lactation and by Production Level

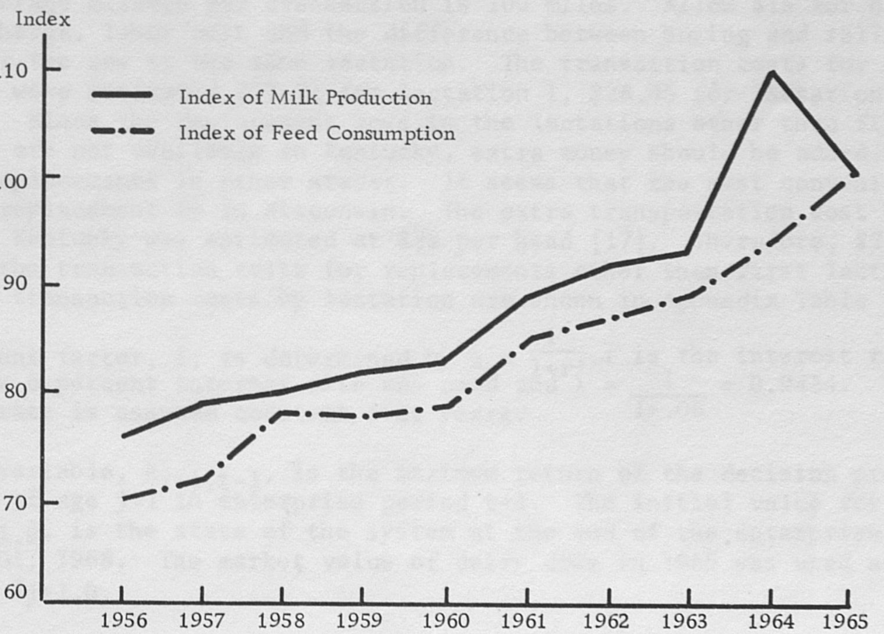

Fig. 2.—Index of Milk Production and Feed Consumption (1965 = 100)

be specified in the following discussion. The first variable is the market value of the present animal, $V_{i,t}$ and is the salvage value of the animal plus the expected net returns of dairy production from present and subsequent lactations. The market value of animals in 1965 was estimated (Appendix Table 5). The market value in other years was adjusted by the price index of dairy cows for each year.

The probability of failure of lactation j,Q_j, was obtained by dividing the number of age j cows that failed within one year by the total number of age j cows in the same year. The probability of success was obtained by $1-Q_j$, where $0 \leq P_j \leq 1$, $0 \leq Q_j \leq 1$. The probability of success is presented in Appendix Table 1 and Fig. 4.

Net return from success, $NS_{j,t}$ was obtained by subtracting feed cost from the market value of milk production. Costs and returns which are constant to animals of all lactations, such as labor, barn, facility charges and value of the calf are not considered here.

Net return from failure, $NF_{j,t}$, is simply the salvage value of the animal. This was obtained by multiplying the price of beef at time t by body weight of the animal.

Market value of the replacement, $V_{j,t}$, was considered in the same manner as the market value of the present animal, $V_{i,t}$. The value of $V_{j,t}$ is assumed equal to the value of $V_{i,t}$, when $i = j$.

The transaction cost, $S_{j,t}$, includes the commission charge, cost of transportation of cows to and from the market, labor cost and any difference between the prices of $V_{i,t}$ and $V_{j,t}$ when $i = j$. The transportation cost of dairy cows in Kentucky is around 12 cents per hundred weight per hundred miles; average mileage per transaction is 100 miles. Allow $15 for commission charge, labor cost and the difference between buying and selling prices for the cow at the same lactation. The transaction costs for each lactation were estimated $27.71 for lactation 1, $28.45 for lactation 2, and so forth. Since the replacement cows in the lactations other than first lactation are not available in Kentucky, extra money should be added for finding the replacements in other states. It seems that the most convenient market to buy a replacement is in Wisconsin. The extra transportation cost from Wisconsin to Kentucky was estimated at $25 per head [17]. Therefore, $25 was added to the transaction costs for replacements other than first lactation. Values of transaction costs by lactation are shown in Appendix Table 8.

The discount factor, λ, is determined by $\lambda = \frac{1}{1+r}$, r is the interest rate. In this study 6 percent interest rate was used and $\lambda = \frac{1}{1+.06} = 0.9434$. The interest rate is assumed constant over years.

The last variable, $\Pi_{j+1,t-1}$, is the maximum return of the decision process for a cow of age j+1 in enterprise period t-1. The initial value for this term, $\Pi_{j+1,0}$, is the state of the system at the end of the enterprise, December 31, 1965. The market value of dairy cows in 1965 was used as the value for $\Pi_{j+1,0}$.

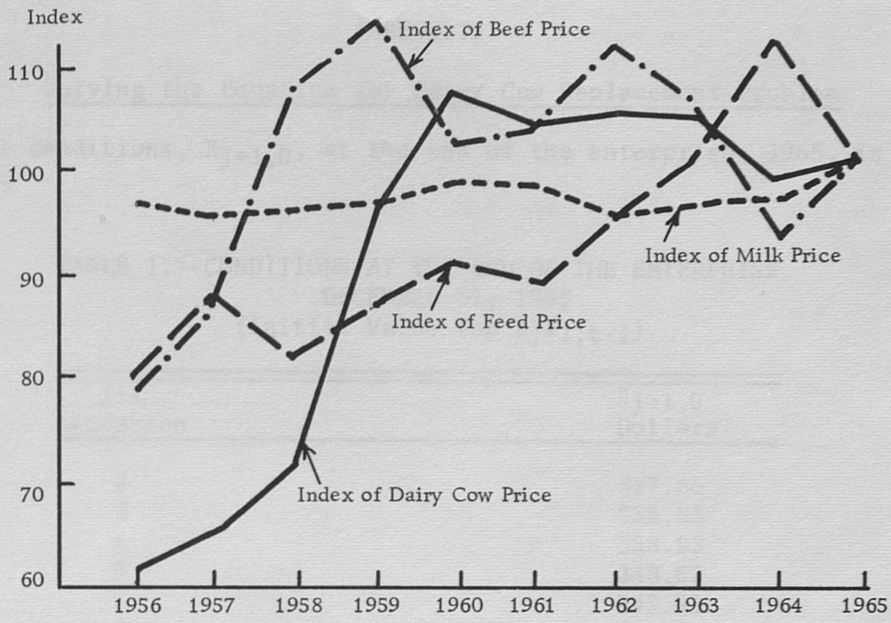

Fig. 3.—Price Indexes of Dairy Cow, Feed, Beef and Milk (1965 = 100)

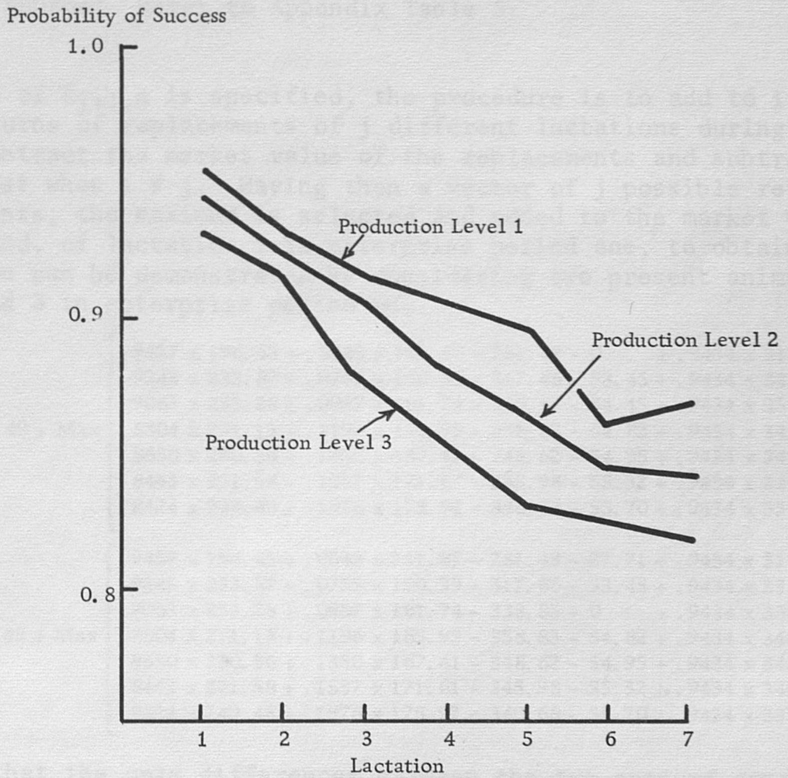

Fig. 4.—Probability of Success of Dairy Cow by Lactation and Milk Production Level

RESULTS

Solving the Equation for Dairy Cow Replacement Problem

The initial conditions, $\Pi_{j+1,0}$, at the end of the enterprise, 1965, is given in Table 1.

TABLE 1.--CONDITIONS AT THE END OF THE ENTERPRISE DECEMBER 31, 1965 (Initial Value for $\Pi_{j+1,t-1}$)

j+1 Lactation	$\Pi_{j+1,0}$ Dollars
2	317.66
3	338.83
4	358.83
5	348.62
6	345.98
7	340.68
8	337.60

Source: Refer to Appendix Table 5.

Once the value of $\Pi_{j+1,0}$ is specified, the procedure is to add to it the expected net returns of replacements of j different lactations during production period one; subtract the market value of the replacements and subtract the transaction cost when $i \neq j$. Having then a vector of j possible returns from replacements, the maximum is selected and added to the market value of the cows on hand, of lactation i in enterprise period one, to obtain $\Pi_{i,1}$. The computation can be demonstrated by considering two present animals of lactation 1 and 3 in enterprise period one.

$$\Pi_{1,1} = 281.49 + \text{Max}\left\{\begin{array}{l} .9457 \times 196.45 + .0543 \times 141.87 - 281.49 - 0 \quad + .9434 \times 317.66 \\ .9245 \times 233.87 + .0755 \times 150.39 - 317.66 - 53.45 + .9434 \times 338.85 \\ .9063 \times 253.28 + .0937 \times 161.74 - 338.85 - 54.45 + .9434 \times 358.83 \\ .8804 \times 271.13 + .1196 \times 165.99 - 358.83 - 54.82 + .9434 \times 348.62 \\ .8650 \times 280.36 + .1350 \times 167.41 - 348.62 - 54.95 + .9434 \times 345.98 \\ .8443 \times 271.58 + .1557 \times 171.67 - 345.98 - 55.32 + .9434 \times 340.68 \\ .8424 \times 249.48 + .1576 \times 175.92 - 340.68 - 55.70 + .9434 \times 337.60 \end{array}\right\}$$

$$\Pi_{3,1} = 338.85 + \text{Max}\left\{\begin{array}{l} .9457 \times 196.45 + .0543 \times 141.87 - 281.49 - 27.71 + .9434 \times 317.66 \\ .9245 \times 233.87 + .0755 \times 150.39 - 317.66 - 53.45 + .9434 \times 338.85 \\ .9063 \times 253.28 + .0937 \times 161.74 - 338.85 - 0 \quad + .9434 \times 358.83 \\ .8804 \times 271.13 + .1196 \times 165.99 - 358.83 - 54.82 + .9434 \times 348.62 \\ .8650 \times 280.36 + .1350 \times 167.41 - 348.62 - 54.95 + .9434 \times 345.98 \\ .8443 \times 271.58 + .1557 \times 171.61 - 345.98 - 55.32 + .9434 \times 340.68 \\ .8424 \times 249.48 + .1576 \times 175.92 - 340.68 - 55.70 + .9434 \times 337.60 \end{array}\right\}$$

It is noticed that the only differences between the two sets of computations are the market value of the animal on hand and the point at which the transaction cost is 0. The maximum returns from the decision process of cows at age 1-7 at the end of enterprise period one, i.e., $\Pi_{1,1}$, are given as follows:

$$\Pi_{1,1} = \begin{pmatrix} 493.16 \\ 547.24 \\ 583.22 \\ 587.44 \\ 591.51 \\ 577.42 \\ 556.38 \end{pmatrix}$$

These $\Pi_{i,1}$ values are used for the $\Pi_{j+1,1}$ value ($\Pi_{j+1,t-1}$) in calculating the returns from the j possible replacements in enterprise period two; the maximum of these returns is added to the market value of the cow on hand of lactation i, resulting in $\Pi_{i,2}$. The same type of iteration is made for each enterprise period until t equals 10. The following example is the computation for $\Pi_{1,4}$, i.e., a present animal of lactation 1 in enterprise period four.

$$\Pi_{1,4} = 294.16 + \text{Max} \begin{pmatrix} .9457 \times 183.14 + .0543 \times 151.62 - 294.16 - 0 \quad + .9434 \times 975.97 \\ .9245 \times 216.48 + .0755 \times 160.72 - 331.95 - 53.45 + .9434 \times 1008.79 \\ .9063 \times 234.36 + .0934 \times 172.85 - 345.10 - 54.45 + .9434 \times 1015.46 \\ .8804 \times 250.27 + .1196 \times 177.40 - 374.98 - 54.82 + .9434 \times 1016.03 \\ .8650 \times 258.44 + .1350 \times 178.91 - 364.31 - 54.95 + .9434 \times 1003.60 \\ .8443 \times 251.14 + .1557 \times 183.46 - 361.55 - 55.32 + .9434 \times 958.20 \\ .8424 \times 232.23 + .1576 \times 188.01 - 356.01 - 55.70 + .9434 \times 924.35 \end{pmatrix}$$

This set of computations shows how the value of $\Pi_{j+1,t-1}$ has been accumulated over three enterprise periods plus the initial conditions.

Interpretation of Results

The optimal replacement policies were determined by using actual prices for the enterprise periods 1956-65 for each of three production levels. The optimal policy for production level 1 is 3, 4, 5, 6, 1, 2, 3, 4, 5, 6. This policy is read as follows: The enterprise begins with animals of lactation three at the beginning of 1956. In the second enterprise period these cows will be in lactation four, and the optimal policy indicates replacement with cows of lactation four in the second enterprise period. Hence, these same cows are kept for another year. At the third enterprise period, cows of lactation five are on hand and at this time the policy determines optimal replacement with cows of lactation five. The same group of cows are kept until the end of the fourth enterprise period. Then the cows of lactation seven are replaced with cows in the first lactation. At the next enterprise period the animals on hand are in second lactation, and the policy dictates replacement with second lactation cows. The policy for the following periods, in order, are third, fourth, fifth and sixth lactation; therefore, through the end of the enterprise no cows will be replaced. The replacement cycle for the optimal policy is six years. The reason that the enterprise starts with cows in the third lactation is due to the life span of the enterprise. If 12 years had been used for the length of the enterprise life span, which would be two complete replacement cycles, the enterprise should start with cows in the first lactation instead of the third.

The optimal policy for production levels 2 and 3 are 3, 4, 5, 6, 1, 2, 3, 4, 5, 6 and 3, 4, 5, 6, 2, 3, 4, 5, 6, 7, respectively. They are interpreted in a manner similar to that for production level 1. The results show that the optimal policies for production level 1 and 2 are identical, while for production level 3, it is more profitable to keep the cows one year longer in the herd. Since the policy outcome depends to a large degree on milk production by different lactation, i.e., the shape of the production curve, one may refer back to Fig. 1, milk-production curves by lactation, to note the reason for the different policies. In Fig. 1, the production curves by lactation for production level 1 and 2 are almost parallel, while that for production level 3 is not parallel to the other curves. The difference between the young cows is less than between the old ones. Therefore, it is quite reasonable that an identical policy may be had for level 1 and 2, while for level 3 the cow should be kept one year longer.

Prices and other parameters used for the replacement-decision process have certain effects on the outcome of the decision. Each parameter may affect (1) the optimal replacement policy, (2) the maximum revenue from the enterprise, or (3) both.

Several values of each parameter are used to examine the effects of the parameter. However, only cows of production level 2 are used for illustrative purposes. Prices used are averages of 1956-65 and various combinations of increase and decrease.

Two groups of transaction costs are used. The transaction E_1 used in this study is shown in Appendix Table 8. The transaction cost without adding extra money for delivering a cow from other states is E_2. Interest rates of 6, 10, 15, and 20 percent are used. The results are represented in Table 2 and the condition of each case, which is different from the standard one, is described. The standard condition is that prices are constant at 1956-65 averages, E_1 and 6 percent interest rates are used.

The most important consideration with respect to the replacement decision is the difference in profitability between the replaced animals and the replacements. Only when the change of parameter values alters the range of value of variables for different lactations in the equation, and not the absolute value for the variables, is the optimal policy affected. However, either a change in range or absolute value of the variables will influence returns from the enterprise. Effects of variations in parameter values are summarized as follows (see Table 2):

1. The optimal replacement cycle for the standard condition was six years. If the length of enterprise life span includes complete replacement cycles, the policy is to read: obtain animals of lactation one, keep until the end of the sixth lactation and then replace with an animal of lactation one.

2. Changes of interest rate will not influence the policy until interest

TABLE 2.--OPTIMAL REPLACEMENT POLICIES AND RETURNS FOR DAIRY COWS IN PRODUCTION LEVEL 2, UNDER VARIOUS CONDITIONS

Conditions	Replacement Policy	Returns
1. Standard	1,2,3,4,5,6	
2. Interest rate at 6 10, 15 percent	1,2,3,4,5,6	Decrease as interest rate increases
3. Interest rate at 20 percent	1,2,3,4,1,2	Decrease
4. E_2	3,4,5,2,3,4	Increase
5. Milk price increase 20 percent	1,2,3,4,5,6,7	Increase
6. Milk price decrease 20 percent	1,2,3,4,5,1	Decrease
7. Feed price increase 20 percent	1,2,3,4,5,1	Decrease
8. Feed price decrease 20 percent	1,2,3,4,5,6,1 or 7	Increase
9. Beef price increase 20 percent	1,2,3,4,5,6	Increase
10. Beef price decrease 20 percent	1,2,3,4,5,6	Decrease
11. Dairy cows price increase 20 percent	1,2,3,4,5,6	Increase
12. Dairy cows price decrease 20 percent	1,2,3,4,5,6	Decrease

rate increases to 20 percent and then has the effect of requiring a more intensive culling policy. However, net return from the enterprise decreases with increasing interest rates. When the interest rate increases, the value for the discount factor, λ, decreases the absolute value of $\lambda\Pi_{j+1,t-1}$ and thus decreases $\Pi_{i,t}$. Because maximum returns from enterprise for cows in different lactations, $\Pi_{j+1,t-1}$, are multiplied by the same value of λ, changes of λ will not influence the range of $\Pi_{j+1,t-1}$ value much and thus has little effect on the optimal policy.

3. The unadjusted transaction cost calls for more intensive culling and replacing with animals in lactations other than first lactation. It is obvious from inspecting the parameters of the equation that without considering the difficulty of finding a replacement of lactation other than 1, the most profitable replacement will never be in first lactation, since the expected net return for the first lactation is much lower than that of the others. The unadjusted transaction cost, E_2, implies a lower value for $S_{j,t}$ and thus increases $\Pi_{i,t}$, the return from the enterprise.
4. A higher milk price or a lower feed price will lengthen the replacement cycle and increase returns from the enterprise. Because when milk price increases or feed price decreases the expected net return from success, $NS_{j,t}$, will increase and the range for $NS_{j,t}$ will also increase ($NS_{j,t}$ = milk price x milk production by lactation - feed price x feed consumption by lactation). Lower milk prices or higher feed prices have opposite effects.
5. A 20 percent increase or decrease in the price of beef effected no change on the replacement policy of dairy cows. Since the beef value, $NF_{j,t}$, is multiplied by a very small value, Q_j, change of beef price cannot have significant effect on the policy. However, the increase of beef price does increase revenue from the enterprise to a small extent. When the beef price decreases, the opposite effect occurs.
6. Changes in the price of dairy cows will not influence the policy. When the price of dairy cows in different lactations increases simultaneously, the price for replaced animals will increase at the same time and have little effect on the policy outcome but will increase returns because the value for the initial state, $\Pi_{j+1,0}$, will increase. A change in the range of dairy cow price for different lactations does have significant effect on the policy in that an increase in the range of prices will shorten the replacement cycle.

Comparison of Replacement Policies Followed by Dairy Producers with the Optimal Policy

The average culling rates for years 1956-65 for both Kentucky dairy herds [18] and Holstein D.H.I.A. herds in Kentucky [19] were 24 percent and 20 percent respectively. While the optimal replacement cycles determined by the replacement model were 1, 2, 3, 4, 5, 6 or 2, 3, 4, 5, 6, 7 implying a culling rate of approximately 17 percent for all production levels. The discrepancy for different culling rates might arise from various reasons. It is most likely that producers' lack information both for production and prices. This could mean that the herd owners have different culling criterion other than profit maximization. However, the culling rate for D.H.I.A. herds was closer to the optimal culling rate than the average in Kentucky. This could imply that the D.H.I.A. herdsmen have more information and a better replacement policy performance in terms of maximum profit.

The expected net returns for dairy herds over the enterprise period is determined by the herd size and net returns per operation unit.[1]

Following the optimal replacement policy, the expected net returns discounted to the value of 1956 for one operation unit through the specified enterprise period is $2,040 while the actual discounted net returns for the D.H.I.A. Holstein herds[2] in Kentucky and the state as a whole[3] are $1,780 and $1,070. These show the closer the replacement policies followed by farmers to the optimal one the higher the net returns will be. Therefore, the hypothesis that the optimal replacement of dairy cows can increase net returns of the dairy farm over the life span of the enterprise is supported.

SUMMARY AND CONCLUSIONS

In a dairy enterprise, the operator spends a considerable proportion of his time and inputs in raising or acquiring replacements for the producing cows. Therefore, the timing of replacement of productive animals is a crucial factor in determining the profitability of his entire enterprise. Determination of a replacement policy is a multistage decision process, and there are many decisions to be made even within one stage. In the conventional approach, each decision may be thought of as a choice of certain numbers of variables which determine the transformation to be employed; each sequence of choices or policy, as shall be said, is a choice of a large set of variables, i.e., a large number of equations have to be solved simultaneously.

Because of the complexity of the problem and the multiplication of dimensions in using a conventional farm management model, it is virtually impossible to determine precisely the replacement policy for a dairy enterprise. Without any formal analysis the farmer is forced to use intuitive judgment in solving his replacement problem. A fundamental problem that confronts a researcher is to devise a new model that will avoid this multiplication of dimension and make the problem more tractable. Dynamic programming is a newly developed mathematical technique, which changes one problem of many dimensions to problems of one dimension.

A recursive equation was introduced in this study to determine the optimal replacement policy for D.H.I.A. Holstein herds in Kentucky for enterprise period 1956-1965. The optimal criterion is the maximum net return over the life span of the enterprise. The term "replacement" here means any case where a new dairy cow is substituted for another one already in the enterprise, whether the replacement is accompanied by a change in output or not. Attention was concentrated on selecting the best lactation period for re-

---

[1] Maintain one cow through the whole enterprise period.

[2] Calculated from [15].

[3] Estimated from adjusting D.H.I.A. herds net returns by state average production.

placement. Another phase of the problem, determining the best time for replacing an animal within the lactation, was not considered. However, one can change the time unit (time interval between decisions) to a month or a week and determine the optimal time for replacement within each lactation with the same procedure used in this study.

The optimal replacement policies determined by using actual prices for the enterprise period 1956-65 for each of the production levels (less than 12,000 pounds, 12,000-15,000 pounds, and over 15,000 pounds) are 3, 4, 5, 6, 1, 2, 3, 4, 5, 6 for production levels 1, and 2 and 3, 4, 5, 6, 2, 3, 4, 5, 6, 7 for production level 3. This means that cows already in their third lactation at the present enterprise period will be kept until the end of the fourth enterprise period which completed their sixth lactation. Then they will be replaced with cows in their first lactation to start the fifth enterprise period and kept in the herd until the tenth enterprise period when the cows reach their sixth lactation. The replacement cycle is six years and the culling rate is 17 percent, which is lower than the actual culling rate for both Kentucky dairy herds and D.H.I.A. Holstein herds in Kentucky. Following the optimal replacement policy, the expected net returns discounted to the value of 1956 for one operation unit through the specified enterprise period is $2,040, which is much higher than the actual discounted net returns for the D.H.I.A. Holstein herds in Kentucky and also the state as a whole. The effects of variations in parameter values upon optimal policy and returns were determined.

There are some limitations of this study which should be noted here. First, the criterion used for determining the optimal policy is profit maximization over the life span of the enterprise. Other criteria could be used for deriving the optimal policy. For example, a replacement policy may be followed which will result in a maximum milk production at all times, or in a minimum labor cost over years, etc. However, these possibilities are disregarded in this study but might be considered in subsequent research.

Second, it was assumed that there will be a market for replacements in every specified lactation. As a practical matter, in Kentucky, no well established market exists for buying replacements other than first lactation. This shortcoming is simply corrected by adding an extra transportation fee to the transaction costs for animals other than first lactation. However, the feasibility of such a market could itself be an interesting topic of research.

Third, genetic improvement over time in a population of dairy cows is not accounted for in the model. However, the index of production is used to adjust production in years other than 1965. It is obvious that there are other variables involved besides genetic improvement in production variation over years. Although the measurement for genetic improvement is simplified, it is adequate for the present study.

Fourth, historical data were used in this study. Little attention was given to the estimation of parameters in the model discussed. Such limitations are necessary to keep the problem within manageable dimensions. However, the control of any problem depends upon the nature of the problem and the stage of research on it. This sort of analytical model is still in its infancy as far as farm management is concerned. There is reason for optimism concerning its potential application in the subject field of farm management, especially at a time when the scale of agricultural production is increasing. It is expected that farmers' adoption of the analytical model will extend that part of the decision dominated by formal analysis and reduce that covered by intuitive judgment.

The success of the application of this approach in actual practice depends, to a large extent, on the accuracy of price forecasts and other estimates for the parameters used. The large dairy industry or a group of integrated producers with similar conditions can apply this model by fitting their own specific data. Programming the equation for electronic computing machines enhances the application for the individual farmer and will speed obtaining results for valid and useful decisions.

REFERENCES

[1] Burt, Oscar R. "Optimal Replacement Under Risk," *Journal of Farm Economics*, Vol. XLVII, No. 2, pp. 324-346, May 1965.

[2] Bellman, Richard E. *Dynamic Programming*. Princeton, New Jersey: Princeton University Press, 1957.

[3] Carlson, Sune. *A Study on the Pure Theory of Production*. New York: Kelly and Millman, Inc., 1956.

[4] Chernoff, Herman and Moses, Lincoln E. *Elementary Decision Theory*. New York: John Wiley and Sons, Inc., 1960.

[5] Day, R. H. "An Approach to Production Response." *Agricultural Economics Research*, Vol. XIV, No. 4, pp. 134-148, Oct. 1962.

[6] Eddison, R. T., Rennycuick, K., and Rivett, B. H. P. *Operational Research in Management*. New York: John Wiley and Sons, Inc., 1962.

[7] Ezekiel, Mordecai, and Fox, Karl A. *Methods of Correlation and Regression Analysis*. New York: John Wiley and Sons, Inc., 1965.

[8] Faris, J. Edwin. "Analytical Techniques Used in Determining the Optimum Replacement Pattern," *Journal of Farm Economics*, Vol. XLII, No. 4, pp. 755-766, Nov. 1960.

[9] Faustman, Martin. "Berechnung des Wertes, Welchen Waldboden, sowie noch nicht haubare Holzbestande fur die Waldwirtschaft besitzen," *Allg. Forst und Jagd Zeitung*, XXV (1849), taken from [3].

[10] Gaffney, M. Mason. *Concepts of Financial Maturity of Timber and Other Assets*. North Carolina State College, A. E. Information Services No. 62 (1957).

[11] Giaever, Harold. "Optimal Dairy Cow Replacement Policies." Unpublished Ph. D. dissertation, University of California, Berkeley, 1966.

[12] Howard, Ronald A. *Dynamic Programming and Markov Processes*. Cambridge, Mass.: The M.I.T. Press, 1960.

[13] Hutton, Robert F., Ihnen, Loren A., Burt, Oscar R., and Morris, W. H. M. "Operations Research Techniques in Farm Management." *Journal of Farm Economics*, Vol. XLVII, No. 5, pp. 1400-1414, Dec. 1965.

[14] Jenkins, Keith, and Halter, Albert. *A Multistage Stochastic Replacement Decision Model*. Agricultural Experiment Station Technical Bulletin 67, Oregon State University, Corvallis (1963).

[15] Preinreich, Gabriel A. D. "The Economic Life of Industrial Equipment." *Econometrica*, VIII (1940).

[16] U. S. Department of Agriculture and Kentucky State Agricultural Extension Services. *Kentucky Dairy Herd Improvement Association Monthly Report*, 1965.

[17] U. S. Department of Agriculture, Economic Research Service. *Cost of Transporting Bulk and Packaged Milk by Truck*. Marketing Research Report No. 791 (1966).

[18] U. S. Department of Agriculture, Economic Research Service. *Supplement to Dairy Statistics*, 1956-1965.

[19] U. S. Department of Agriculture, Agricultural Research Service, Animal Husbandry Research Division. *Dairy Herd Improvement Association Yearly Herd Summary*, 1956-1965.

[20] U. S. Department of Agriculture, Statistical Reporting Service, Crop Reporting Board. *Milk Production and Dairy Products Annual Statistical Summary*, 1966.

[21] U. S. Department of Agriculture, Statistical Reporting Service and Kentucky Department of Agriculture, Kentucky Crop and Livestock Reporting Service. *Kentucky Dairy Supply and Marketing Statistics*, 1965.

[22] Winder, J. W. L., and Trant, G. I. "Comments on Determining the Optimum Replacement Pattern." *Journal of Farm Economics*, Vol XLIII, No. 4, Part 1, pp. 939-951, Nov. 1961.

[23] White, W. Cleland. "A Nonstochastic Approach to the Determination of an Optimal Hen Replacement Policy." Unpublished Masters thesis, University Of Kentucky, 1959.

[24] Yamane, Taro. *Elementary Sampling Theory*. Englewood Cliffs, N. J.: Prentice Hall, Inc., 1967.

APPENDIX

TABLE 1.--PROBABILITY OF SUCCESS OF DAIRY COWS GIVEN THE MILK PRODUCTION LEVEL AND LACTATION

Lactation	Probability of Success		
	Production Level 1 (less than 12,000 pounds/ 305 days)	Production Level 2 (12,000-15,000 pounds/ 305 days)	Production Level 3 (more than 15,000 pounds/ 305 days)
1	.9562	.9457	.9326
2	.9338	.9245	.9165
3	.9175	.9063	.8778
4	.9073	.8804	.8562
5	.8973	.8650	.8322
6	.8607	.8443	.8249
7	.8678	.8424	.8187

Source: Calculated from Pennsylvania D.H.I.A. Data, 1960.

TABLE 2.--ESTIMATED MILK PRODUCTION BY LACTATION AND PRODUCTION LEVEL, 1965

Lactation	Milk Production (pounds/305 Days)		
	Production Level 1[a]	Production Level 2[b]	Production Level 3[c]
1	7,748	10,759	13,246
2	9,256	12,044	14,890
3	10,328	12,997	16,174
4	10,963	13,618	17,097
5	11,162	13,907	17,659
6	10,926	13,864	17,861
7	10,253	13,488	17,702

[a]The standardized production (yields in 305 days for a mature cow, at age 6) is less than 12,000 pounds.

[b]The standardized production is between 12,000 and 15,000.

[c]The standardized production is more than 15,000 pounds.

Source: The estimates are based on data collected from the *Kentucky D.H.I.A. Lactation Report*, U.S.D.A. and University of Kentucky Cooperative Extension Services cooperating.

APPENDIX--*Continued*

TABLE 3.--INDEX OF MILK PRODUCTION
1956-65

Year	Average Production (pounds)[a]	Index[b]
1956	8,304	76
1957	8,574	79
1958	8,694	80
1959	8,996	82
1960	9,082	83
1961	9,666	89
1962	10,093	92
1963	10,151	93
1964	11,297	110
1965	10,921	100

[a]*Kentucky Dairy Herd Improvement Record*, U.S.D.A., Agricultural Research Service, Dairy Cattle Research Branch and State Agricultural Extension Services Cooperating.

[b]Index is obtained by dividing the production in each year into that in year 1965 (1965 = 100).

TABLE 4.--INDEX OF BODY WEIGHT OF DAIRY COWS BY LACTATION

Lactation	Average Weight[a]	Index[b]
1	1,083	100
2	1,148	106
3	1,240	114
4	1,263	117
5	1,277	118
6	1,310	121
7 and over	1,348	124

[a]Calculated from *Kentucky Dairy Herd Improvement Record, Monthly Report*.

[b]Calculated by dividing body weight of each lactation by that of the first lactation (first lactation = 100).

APPENDIX--*Continued*

TABLE 5.--ESTIMATED MARKET VALUE OF DAIRY COWS BY LACTATION AND PRODUCTION LEVEL, 1965

Lactation	Estimated Market Value (dollars)		
	Production Level 1	Production Level 2	Production Level 3
1	225.19	281.49	337.79
2	254.13	317.66	381.19
3	271.08	338.85	406.62
4	287.06	358.83	430.60
5	278.30	348.62	418.34
6	276.78	345.98	415.18
7	272.54	340.68	408.82
8	270.08	337.60	405.12

Source: Data were collected at Blue Grass Stock Yards and Clay-Wachs Stock Yards in Lexington, Kentucky.

TABLE 6.--PRICE INDEXES OF DAIRY COW, FEED, BEEF AND MILK, 1956-65

Year	Price of Dairy Cow Index	Feed Price Index	Beef Price Index	Milk Price Index
1956	61.6	79.6	77.6	96.5
1957	65.0	86.5	85.9	95.0
1958	71.2	81.5	107.3	95.3
1959	94.9	86.5	113.7	96.0
1960	106.8	90.0	102.0	98.0
1961	103.4	88.1	102.9	97.5
1962	104.5	94.2	111.2	94.8
1963	104.0	99.6	104.9	96.0
1964	97.7	111.5	87.8	96.5
1965	100.0	100.0	100.0	100.0

Source: Calculated from the data of Division of Agricultural Price Statistics, U.S.D.A. Bureau of Agricultural Economics, Agricultural Estimates (1965 = 100).

APPENDIX--*Continued*

TABLE 7.--INDEX OF FEED COST

Year	Feed Cost[a]	Adjusted Feed Cost[b]	Index[c]
1956	135	164	70
1957	142	170	72
1958	150	184	78
1959	158	183	78
1960	167	186	79
1961	176	200	85
1962	193	205	87
1963	208	209	89
1964	246	221	94
1965	235	235	100

[a]Obtained from Kentucky D.H.I.A. Yearly Herd Summary.

[b]Adjusted by index of feed price.

[c]Index is obtained by dividing the adjusted feed cost in each year into that of year 1965 (1965 = 100).

TABLE 8.--TRANSACTION COSTS BY LACTATION

Lactation	Transaction Costs
1	27.71
2	53.45
3	54.45
4	54.82
5	54.95
6	55.32
7	55.70

Source: Estimated from data obtained at Blue Grass Stock Yards and Clay-Wachs Stock Yards in Lexington, Kentucky.