

**A MULTIPERIOD LINEAR PROGRAMMING-SIMULATION
MODEL OF THE FARM FIRM GROWTH PROCESS**

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Late-Stage Shifts in Baby Tobacco Allotments

1950-51

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CONTENTS

	Page
Preface	2
List of Tables	3
List of Figures	4
Introduction	5
Concepts Pertinent to Farm Firm Growth Studies	8
The Model: Conceptual Framework	9
The Model: Operational Form	13
The Study Farm	18
Analysis of Results	20
Summary and Conclusions	36
References	39
Appendix A: Multiperiod Linear Programming Model	41
Appendix B: Flow Chart of the Simulation Model	53

PREFACE

This report concerns results of research conducted by the authors at the University of Kentucky in 1971 and 1972 and are based in large part on the work conducted by senior author as a part of his Ph.D. dissertation.

The authors contend that prior to undertaking this investigation that considerable effort, with varying degrees of success, had been expended by agricultural economists in developing and using farm firm growth models of either of two general types: (1) optimizing (mostly LP, etc.), or (2) descriptive-accounting (mostly computer simulation). The primary thrust of our investigation was not to refine the two techniques or approaches but, rather, to link the best features of the two into a single, computer-operational model. Thus, the first portion of the report is devoted primarily to conceptual and theoretical matters-literature review, constructing the combination model and discussing its features. The second portion is devoted largely to empirical matters-testing, using and experimenting with the model on a Central Kentucky beef cattle farm.

LIST OF TABLES

Table	Page
1. Estimates of Initial Expected Values for Yields and Prices, and Coefficients of Expectation	14
2. Farm Enterprise Organization and Financial Situation of the Case Farm, January 1967 and December 1970	19
3. Farm Enterprise Organization Generated from the Growth Model	25
4. Summary of Simulated Financial Outcomes for the Study Farm	27
5. Means and Variations of Total Assets and Net Worth as Simulated by the Growth Model	28
6. Comparison of Simulated Farm Enterprise Organizations, Three Alternative Production Systems on the Study Farm 1971 through 1974	31
7. Output Data Generated By the Growth Model for Net Worth at the End of 1974 . . .	34
8. Output Data Generated By the Growth Model for Total Assets at the End of 1974 . .	34
9. Statistics for Analysis of Variance for Total Assets at the End of 1974	35
10. Statistics for Analysis of Variance for Net Worth at the End of 1974	35
 Appendix A	
Table 1	
A Two-Year Version of the Multiperiod Linear Programming Tableau for the Study Farm	47

LIST OF FIGURES

Figure	Page
1 Conceptual Process Model of the Farm Firm Growth	12
2 Actual and Simulated Mean Values of Total Assets at End of Year	23
3 Actual and Simulated Mean Values of Net Worth at End of Year	23
4 Actual and Simulated Mean Number of Beef Cows in the Herd at End of Year	24
5 The Projected Time Paths for Total Assets Under the Three Operating Alternatives . . .	32
6 The Projected Time Paths for Net Worth Under the Three Operating Alternatives . . .	32
Appendix B	
Figure 1	
Flow Chart of the Simulation Model	53

age
12
23
23
24
32
32
53

A MULTIPERIOD LINEAR PROGRAMMING-SIMULATION MODEL OF THE FARM FIRM GROWTH PROCESS

by

Ying I. Chien and Garnett L. Bradford*

During the past three decades one of the most dramatic changes in U.S. agriculture has been the continuous trend toward fewer and larger farms. This revolution has been due mostly to the adoption of new production technology and the substitution of capital inputs for labor. Changes in market and farm structure have brought about the necessity for farm businesses to be large enough to remain competitive and to earn ample income for family needs.

The Problem

This trend toward fewer and larger farms is likely to continue, even if at a slower rate. Accompanying the trend, are a number of questions about which farmers, agribusinessmen, policy makers, and researchers are concerned. Pertinent examples are: (1) What growth patterns of the farm firms are to be expected when farmers contemplate alternative farm management strategies? (2) As a farm business expands, will greater reliance on borrowed funds

become necessary for the acquisition of additional resources because self-owned and/or internally generated capital will likely be insufficient for expansion over time? Consequently, will farmers and capital lending agents be interested in quantifying the potential for growth of the farm firm under different capital market structures? (3) How may the impact of the variability of such factors as prices and yields be expected on the firm growth potential? Information pertaining to these and other related questions is necessary for future farm adjustment.

To answer such questions, fruitful models of the farm firm growth process need to be developed. Farm firm growth models which were most popular or most often employed during the 1960's may be classified into three types: multiperiod linear programming, recursive linear programming, and simulation models.¹ Of these three, the multiperiod linear programming (MLP) technique has been the most widely employed. Although it has been considered to be an efficient and flexible tool for analyzing farm firm growth problems, its solution still remains questionable vis-a-vis the reality of

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¹For a more complete review of the applications of these models to farm firm growth, see Irwin [1968].

actual farm operations. Critical shortcomings which have been cited may be outlined:

- (1) The MLP model, as its name implies, explicitly provides a simultaneous solution for all periods in the planning horizon. It does not, therefore, account for the sequential nature of production or marketing decisions. Consequently, the farm firm growth process generated by this kind of model may not be compatible with the real world setting in which farms typically operate.
- (2) Multiperiod linear programming is an optimizing model, so it results in a global optimum by solving the system of the model for all periods simultaneously. This characteristic may render the model to be less satisfactory when one is interested in descriptive or predictive economic analysis rather than "prescriptive" (making recommendations for resource or output adjustments).
- (3) This model, like the standard linear programming approach, carries the assumption that all inputs and products are perfectly divisible. Clearly, for most farm firms many items come in indivisible units. For example, a farmer can not buy a 0.75 unit of tractor or sell 0.5 head of beef cattle.
- (4) Multiperiod linear programming is a dynamic-certainty model; perfect knowledge about input-output and price coefficients is assumed to exist over all periods. The assumption of certainty usually implies the notion that expectations of the future are single-valued and correct and, hence, renders the model to be inadequate and unrealistic. This assumption can, however, be relaxed, e.g., S. R. Johnson, et. al. [1967] have employed the Monte Carlo Method to allow crop yields to vary in his multiperiod linear programming model.
- (5) Multiperiod linear programming model cannot (by itself) adequately handle and predict financial variables such as total assets or total net worth.
- (6) It is difficult to incorporate "qualitative" factors into an MLP model. For example, it seems reasonable to assume that farmers' decision rules for asset investment depend not only on easily quantified economic factors but also on the necessity of assets in terms of timeliness of asset services.

The recursive linear programming approach was first employed by T. Heidhues [1966] to solve growth problems for farm firms. This approach deals with the dynamics of decision-making by using a sequential optimizing procedure to describe how plans for a given time period are related to past expectations and performance. Consequently, it is applicable to a wide variety of dynamic problems in the field of *positive* economic analysis.

Since recursive linear programming approach employs conventional linear programming to solve problems sequentially, it suffers the drawback of the divisibility problem and the incapability of adequately handling and predicting financial variables. In any event, it seems that this approach is not entirely satisfactory for application to firm growth studies, because expectations about future conditions and possible outcomes in the future periods are not taken into explicit consideration.

Computer simulation becomes a relevant and useful tool for analyzing firm-growth economic problems when the researcher faces one or more of the following situations [see Naylor, 1971, pp. 6-9].

- (1) It may be either impossible or extremely costly to observe actual behavior or processes of an economic system. For example, certain data on farm firm growth patterns under different

conditions (e.g., the firm's investment policy, resource situations, etc.) simply do not exist. Computer simulation could be an effective means of generating data which can describe possible growth patterns of the firm.

- (2) The system in study may be so complex that it is impossible to describe it mathematically in such a way that analytical solutions may be obtainable and single-valued predictions can be made. Many decisions made by farm operators fall into this category.
- (3) While some aspects of the system(s) of interest may be describable in a mathematical model, one may not be able to obtain a solution to the model by analytical techniques. Computer simulation methods have been demonstrated to be efficient techniques of numerical analysis for solving complex mathematical problems and stochastic models.
- (4) Conducting experiments to test the validity of mathematical models which describe the behavior of the system may be impossible or too expensive. For example, it is difficult to conduct experiments with actual farms to examine the effects of different production alternatives on the farm firm growth. However, experimentation on the computer provides researchers an efficient tool to handle problems of this sort.

In addition to the foregoing four situations where computer simulation has potential, it has been pointed out that this technique is also useful and appropriate in handling multiple goals, indivisibilities, sequential decisions within the planning period, concepts of organizational, managerial and behavioral theories [see Halter and Dean, 1965 and Hutton, 1966]. Recognizing the relevance of computer simulation technique

to economic analyses, G. F. Patrick and L. M. Eisgruber [1968] first applied this tool to a farm firm growth study, followed by H. R. Hinman and R. H. Hutton [1971] as well as by H. D. Hall and O. L. Walker [1970].

In spite of its potential as a tool for analyzing many economic problems, computer simulation, when applied to dynamic farm planning or farm firm growth studies, also has several deficiencies. This is primarily because simulation models lack linkage in over-all farm planning for each time period within the planning horizon. They typically provide purely a sequential rather than a simultaneous solution to the farm firm growth problem. Yet any given period of time, the farm operator seems likely to draw a blueprint (at least in his mind) for an over-all (simultaneous) farm planning over a number of years. The formulation of this dynamic farm planning is based on the operator's expectations which, as stated in the subsequent section, plays an important role in planning.

Objectives of the Study

Out of the examination of the past farm firm growth models grew the awareness that a research effort should be directed toward developing a farm firm growth model which incorporates the "best" features of these models. Thus, the primary objectives of this report deal with research techniques. More specifically, these objectives are:

- (1) To structure a farm firm growth model which is in conformity with economic theory and consistent, to the extent possible, with reality.
- (2) To develop and apply, based on the conceptual model noted in objective (1), operational farm firm growth models which are capable of depicting and analyzing the farm firm growth process.

To achieve these objectives, certain criteria for the model-building must be met. First, the model should be, in some sense, dynamic and stochastic. That is, account should be made of the fact that firm growth does not take place in an environment of certainty and statics. Second, research effort should be directed toward constructing a *descriptive* model so that the farm firm growth process may be depicted and analyzed. Finally, the model should be operational in the sense that obtainable by means of mathematics and/or computer operations. Accordingly, the MLP-simulation model discussed in this report is applied on a test or preliminary basis to a Central Kentucky Beef Cattle Farm. This application is discussed in the concluding sections.

CONCEPTS PERTINENT TO FARM FIRM GROWTH STUDIES

Before the model is presented, certain definitions and concepts of farm firm growth will be delineated. The terms—farm firm and firm growth—are defined in this section, and the dynamic nature of farm firm growth is briefly discussed.

Definitions

Firm-household interrelationships are not to be ignored. Thus, a "farm firm," as defined in this report, is a business entity which is primarily concerned with the creation of net returns and the satisfaction of certain levels of family living by means of producing agricultural products. From this definition, it follows that the farm firm is a decision making unit on both the production side and consumption side.

The concept of firm growth can be illusive. At present there is no widely accepted unique definition. Firm growth, in

general, is thought of as an increase in the firm size which may be measured in terms of any number of several variables, e.g., volume of output, quantity of resources, and magnitude of accumulated worth, etc. However, the fact that several or all these variables may not change in the same direction—some may increase and some others decrease—presents difficulty in measuring the net change in the size of the firm.

In this report, firm growth is measured primarily in terms of three variables: (1) an increase in total assets, (2) an increase in net worth (equity), and (3) an expansion in the particular productive enterprises (e.g., beef cow herd size). There are several reasons for choosing these three variables as measures of firm growth. The use of total assets allows one to recognize the fact that ultimately firm growth must arise from the acquisition of additional assets to meet the need for expanding the farm business. Change in net worth is a reasonable base of evaluation when we examine growth of the firm as a whole. A change in net worth reflects increases in assets and/or decreases in liabilities. The level of and the change in net worth are, therefore, good indicators of growth capacity of the firm. Consideration of change in the size of major farm enterprises (such as beef cows) is particularly appropriate when financial variables fail to reflect the growth of the farm firm.

The Dynamics of Firm Growth

Like feeding of livestock and raising of crops, the growth process of a farm firm does not take place in a static environment. In the actual growth process, the firm's actions for decision and planning at any given period of time obviously are interrelated closely both with the past and the future. It is, therefore, essential that research on firm growth somehow account for its dynamic nature.

It is because entrepreneurs or farm operators can never be very certain about the future that expectations play an important role in planning. The fact that expectations, and hence planning, may be in error emphasizes the importance of the recursive processes of learning and obtaining information which greatly influence the formulation of expectations. Such processes of learning and obtaining information reveal that expectations stretch over time.

While the firm manager may stick to the same expectations over a certain period of time, it is likely that new expectations arise as time goes by. Accompanying each new experience, expectations are changing and, hence, different at every successive point of time. In other words, the process of learning and obtaining information may be considered to be continuous. As time goes by, the evidence changes continuously as more facts become known and, hence, the prospect may be changing over time accordingly.

Since expectations in one period relative to economic and environmental conditions in future periods might be held with great uncertainty, the production and investment plans which are based on expectations must continuously be adjusted or revised with time. Adjustment of production and investment plans toward "desired" or "optimal" levels should continue with time as knowledge is gained over time. However, owing to various reasons such as lack of knowledge, durability of capital inputs, uncertainty of prices and technology, the firm may carry out adjustments slowly and gradually. In other words, there may exist lagged adjustment in response to changes in the economy and environment.

The dynamic nature of firm growth also can be, in addition to that inherent with expectations, recognized from the fact that decisions concerning production and investment are closely interdependent as the firm grows over time. In other words, business

and financial management overlap. It is quite obvious that the rate of firm growth depends heavily upon the firm's capacity of production which may be expanded through the acquisition of additional resources. The ability of acquiring resources at any period of time is largely dependent upon the availability of funds which may be obtained from both internal and external sources, viz., disposable income in excess of needs for consumption, a tolerable amount of saving and funds borrowed from loan agencies. When the availability of factors of production is not in the form of fractional sized units, the timing of investment becomes important to the firm. Investments in buildings, machinery and equipment serve as an example. When capital is limited, such lumpiness of many inputs complicates investment planning which certainly affects production choices and, hence, the rate of growth.

THE MODEL: CONCEPTUAL FRAMEWORK

The conceptual model of the farm firm growth process rests upon two theoretical notions, viz., Hick's notion [1946] and that of Modigliani and Cohen [1961] regarding the dynamic planning behavior of the firm. Pragmatically, it takes into consideration that the model should be capable, at least to some extent, of "implementing" farm operations into a "real-world setting." A brief review of these two notions is presented in the following subsection.

Theoretical Background

In his "Value and Capital," Hicks [1946] developed a dynamic decision-making model of the firm under certainty. According to his view, just like in static theory, the firm is to choose from among alternative available courses of action the one which is most

conducive to the achievement of its goal. Hicks [p. 193] expressed his idea by arguing that "... the decision which confronts any particular entrepreneur at any date ... may be regarded as the establishment of a production plan." He comments further [p. 194] that "... just as the static problem of the enterprise is the selection of a certain set of quantities of factors and products, so the dynamic problem is the selection of a certain production plan from the alternatives that are open." From this proposition, he concludes that the decision problem faced by the firm at any given point of time is the selection of the best plan over the planning horizon. The most fundamental way of selecting the preferred production plan involving costs and returns in future periods is that of the capitalized value of the stream of surplus--Hicks called it "their capitalized value of the production plan." In establishing this criterion, he [pp. 194-95] contended that:

In statics, we were content to think of the entrepreneur maximizing his surplus of receipts over costs; this caused no special difficulty. But when the problem is looked at dynamically, it becomes clear that the entrepreneur can expect, not a single surplus, but a stream of surpluses, going on from week to week. If two streams were such that every surplus in the one stream was greater than the corresponding surplus in the other stream, then there would be no question which stream was the larger. But if this condition is not fulfilled (and there is no reason why it should be fulfilled always, or even often), we used some criterion to enable us to judge whether one stream is to be reckoned larger than another.

The implication of the Hicks' formulation is that at the beginning of each

period the firm is supposed to face with the maximization problem subject to certain constraints over some definite horizon. It also implicitly indicates that definite estimates of parameters associated with these constraints are obtainable through anticipations.

F. Modigliani and K. Cohen [1961] contend that in reality "economic men" do not generally behave expressly in the way implied by the Hicks' concept. They indicate that the discrepancy between the conclusion of Hicks' analysis and observed behavior may be explained in two ways. First, uncertainty is involved in the real world, and the existence of uncertainty tends to shorten considerably the horizon over which it is useful to form anticipations or to formulate plans. Secondly, Hicks' model which assumes rational behavior on the part of business firm cannot adequately account for the actual behavior. Modigliani and Cohen argue further that "while it is perfectly true that in terms of the pay-off function the single current move of the static model is replaced by the entire set of moves over the horizon, it does not necessarily follow that, as in the static model, the firm must choose now its entire course of action." Therefore, they propose [p. 20]:

... the decision problem confronting the entrepreneur at a given point of time is most usefully regarded not as that of selecting the best possible plan of operations over the horizon, but rather, as that of *selecting the best possible first move only*.

According to Modigliani and Cohen, the "best possible" not only refers to the first period but refers to the entire maximization problem over the horizon. In this sense, this formulation is similar to the Hicks' notion. However, there is conceptual difference in that their formulation places major stress on the *first* period (or sub-period) of the

planning period. In other words, it emphasizes on the choice of the *first move*² which cannot be postponed and, hence, must be made at a given point in time. This approach treats the later sub-periods of the planning period in much less detail than the first sub-period. It also stresses the point that the decision-maker will have to revise his plans when information becomes available through time, even if he has great confidence. Long-run plans are, therefore, not necessarily made up in order to be implemented, but only to utilize all the available information to make the best plan for the current period.

Construction of the Conceptual Model

As sketched in Figure 1, a general conceptual model of farm firm growth process may be described in three phases:

- (1) Formulation of expectations and the *ex ante* long-run plan,
- (2) Implementation of the current mono-periodic plan, and
- (3) Reformulation of expectations and the *ex ante* long-run plan.

At any given period of time the manager is, in Hicks' concept, assumed to have in mind a long-run plan; that is, over a certain planning horizon, he plans for his goals to be achieved. This plan is formulated on the basis of single-valued expectations about prices and yields. This plan is, however, not likely to be carried out for the entire planning horizon because expectations, being a "subjective matter," are subject to errors. As experiences are gained and new information becomes available, expectations are changed over time. Accordingly, the long-run plan must be

revised. From this argument, it is therefore, reasonable to assume that the farmer formulates, at the beginning of each period, a long-run plan with the aim only of providing himself with a basis for farm operation for the current year. In terms of the Modigliani and Cohen's notion, the farmer may be supposed to try to get *the best possible first move* which cannot be postponed and, hence, must be carried out at a given point of time. We may, therefore, call this kind of long-run plan an *expected* or *ex ante* plan.

Having formulated an *ex ante* plan in his mind, the manager then takes action to implement it for the current period (year). Actual outcomes may turn out to be significantly different from the prospects, due to various reasons. The mono-period plan (derived from the *ex ante* long-run plan) may be, due to various factors such as psychological inertia and institutional factors, adjusted during the course of implementation. Actual prices and yields may deviate significantly from the expected levels. The actual investment decision process, at a given point in time, may not be the same as that decision made in the long-run planning. Such phenomena, however, may be incorporated in a research model by employing computer simulation techniques and the concept of behavioral (or flexibility) constraints.³ One of the merits of computer simulation is that the model can be designed to allow for stochastic variables. Thus, when stochastic elements are involved in the model, repeated trials (replications) are made possible by the computer simulation technique so as to average out the effects of "unusual happenings" occurring for any single replication.

At the end of the production period, the current mono-periodic plan becomes a

²Modigliani and Cohen [p. 16] called the action carried out by the firm in any period its *move* in that period.

³The idea is originally due to Henderson (1959, pp. 242-260).

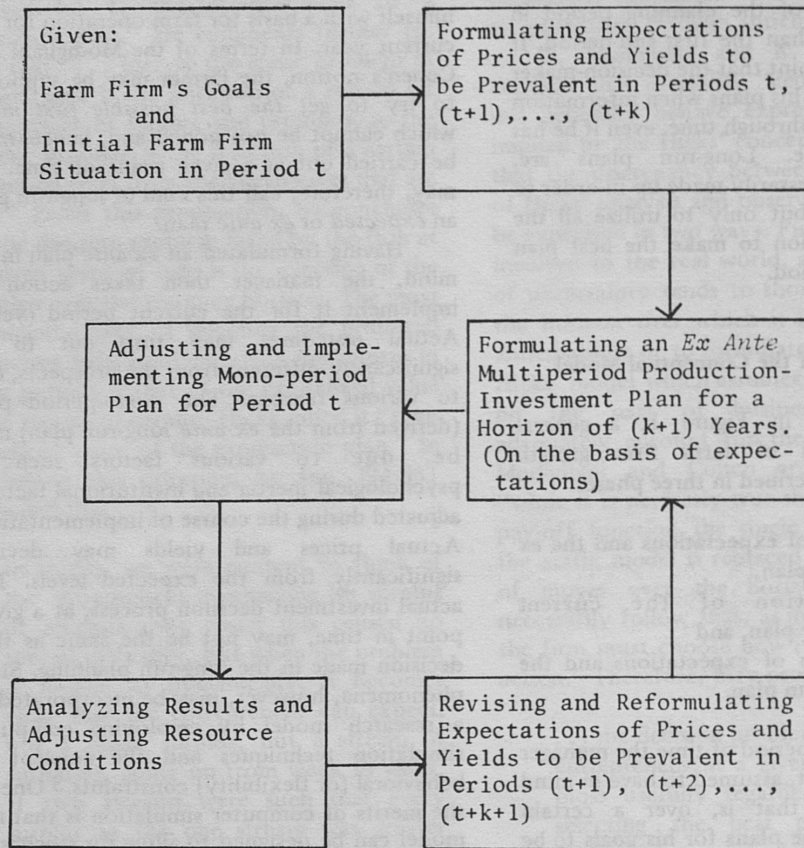


Figure 1. Conceptual Process Model of the Farm Firm Growth

realized or *ex post* plan. The farmer has gradually accumulated knowledge and experiences, and then must revise his expectations and adjust or reformulate another *ex ante* long-run plan over a new horizon.⁴ Repeating the process year after year, a time path of farm firm growth may be traced out.

THE MODEL: OPERATIONAL FORM

Expectation Model

In this study, farmers' expectations about prices and yields are hypothesized by using Nerlove's expectation model [1958]. The basic idea underlying this model is that people's notions of the expected "normal" level of economic variables, such as prices, is affected by their actual past values. However, for any given manager (decision maker), past levels of economic variables do not necessarily exert their influence equally. In other words, more recent prices are a partial result of forces expected to continue to operate in the future; the more recent the price, the more it is likely to express the operation of forces relevant to "normal" levels [Nerlove, 1958, p. 52]. Mathematically, the model may be expressed as:

$$Z^*_t = Z^*_{t-1} + \beta [Z^*_{t-1}]$$

$$0 < \beta < 1 \quad (1)$$

where

Z^*_t is the expected "normal" level of a variable (e.g., price or yield) during period t ,

Z_t is the actual level of the variable during period t ,

β is the coefficient of expectation.

Equation (1) may be rewritten as:

$$Z^*_t = \beta Z_{t-1} + (1 - \beta) Z^*_{t-1} \quad (2)$$

Expanding equation (2) by iteration gives the following expression:

$$Z^*_t = (1 - \beta)^t Z_0 + \sum_{i=1}^t \beta (1 - \beta)^{i-1} Z_{t-1} \quad (3)$$

where

- i denotes the most recent time period (year) for which levels of the variable (Z) were known, and
- t denotes the "current" time period (year).

The nature of this model is readily apparent from equation (3). Given the coefficient of expectation, β , the weights decrease as the actual data get older. It also implies that Nerlove's model is a type of adaptive, error-learning behavior of expectations. It is the nature which makes the model appeal to this study. Presumably, farmers base expectations on their memories. The further one goes back in time, the more vague his memory should be. Therefore, the weights [the $\beta(1 - \beta)^{i-1}$ coefficients decline as data get older.

The initial expected values for yields and prices (Z^*) are 5-year averages (1961-65), computed from *Costs and Returns* [USDA, 1950-1965] and from *Prices of Products Bought and Sold by Kentucky Farmers* [Card and Koepper, 1970]. However, the average yield of corn was adjusted upward by 20 percent to make it more consistent with the study farm records (Table 1).

Expectation coefficients for yields and prices are assumed to be 0.7 and 0.9, respectively. On the basis of commonly accepted "thumb rules" and *a priori*

⁴A new horizon is now extended into a further production year, but the current year is excluded because it is no longer relevant.

TABLE 1

ESTIMATES OF INITIAL EXPECTED VALUES FOR
YIELDS AND PRICES, AND COEFFICIENTS OF EXPECTATION

Item	Unit	Initial Expected Value	Coefficient of Expectation
		Z_0^*	β
Yield			
Corn	Bu.	88.0	0.7
Hay	Ton	2.8	0.7
Price			
Corn	\$/Bu.	1.2	0.9
Feeder calves	\$/cwt.	25.7	0.9
Feeder cattle	\$/cwt.	20.8	0.9

- (1) The estimates of initial expected values for yields of corn and hay were a five-year average (1961-1965) calculated from *Costs and Returns* by ERS, U.S. Department of Agriculture.
- (2) The estimates of initial expected values for prices of corn and livestock were a five-year average (1961-1965) calculated from *Prices of Products Bought and Sold by Kentucky Farmers* by D.G. Card and M. Koepper [1970].

knowledge, it seems reasonable to argue that from the viewpoint of farmers, yields are more controllable than prices. Therefore, farmers' expectations about prices in a particular year should be more dependent upon the actual price levels of less-distant prior years.

Ex Ante Multiperiod Planning Model

Multiperiod linear programming (MLP) is the technique employed in this study to approximate farmers' long-run planning behavior. This does not necessarily mean that farmers formulate and directly use this model. What the application of the model implies is that the representative farmer behaves *as if* he made his long-run plan consistent with the models used by economists. The mathematical formulation of multiperiod linear programming model is presented as follows:

$$\text{Maximize } C(t+i)X(t+i) \quad \text{for } i=0, 1, \dots, k$$

subject to

$$\begin{bmatrix} A(t) \\ A(t+1) \\ \vdots \\ A(t+k) \end{bmatrix} \begin{bmatrix} X(t) \\ X(t+1) \\ \vdots \\ X(t+k) \end{bmatrix} \leq \begin{bmatrix} B(t) \\ B(t+1) \\ \vdots \\ B(t+k) \end{bmatrix} \quad (4)$$

where

$C(t+i)$ is a row vector of expected "normal" net returns from a unit of alternative activities in production period $(t+i)$. The net return from each activity is obtained by subtracting its cash production expenses from the expected "normal" price. This vector

is extended over the planning horizon of $(k+1)$ years.

$X(t+i)$ is a column vector consisting of subvectors, $X(t), X(t+1), \dots, X(t+k)$. These subvectors contain alternative activities in the production years of $t, (t+1), \dots, (t+k)$, respectively.

$A(t+i)$ is a matrix composed of $A(t), A(t+1), \dots, A(t+k)$ which are matrices containing coefficients of transition and input-output transformation for production years of $t, (t+1), \dots, (t+k)$, respectively.

$B(t+i)$ is a vector composed of $B(t), B(t+1), \dots, B(t+k)$ which are vectors of the amounts of resources available for the production years, $t, (t+1), \dots, (t+k)$, respectively.

This model is very similar to conventional linear programming framework with the exception that this model is extended over a number of years. This multi-period programming model is dynamic in a Hicksian sense as prices, inputs, and outputs are dated. The dynamic nature of the model is also characterized by the inclusion of transfer activities in the model, allowing income and surplus resources in one period to be transferred to the subsequent period.

A detailed, real-world description and discussion of the MLP model is rendered in Appendix A. Activities are defined and discussed; constraints are defined and discussed; and the objective function is empirically specified. All this is in terms of the beef cattle study farm discussed in the latter two sections of the report.

The structure of this MLP model is somewhat different from the usual one employed in previous studies, such as [Johnson et.al., 1967; Martin and Plaxico, 1967]. This may be seen from the operation of a "feedback" system to provide information on resource availabilities for each production year. Mathematically, the

"feedback" system may be depicted as follows:

$$B(t) = \Lambda_{t-1} B^*(t-1) + \Gamma_{t-1} B(t-1)$$

for $t = 1, 2, \dots, T$ (5)

and

$$B(t+i) = \Gamma_{t+i-1} B(t+i-1)$$

for all $i, i = 1, 2, \dots, K$ (6)

where

Λ_{t-1} is a diagonal matrix containing rates of depreciation for inputs purchased and used in year (t-1). The rate of depreciation for nondepreciable inputs is one (1).

Γ_{t-1} is a diagonal matrix containing rates of depreciation for all inputs, except those purchased in year (t-1). Again, the rate of depreciation for nondepreciable inputs is one (1).

$B^*(t-1)$ is a column vector in inputs purchased in year (t-1). It is a zero vector at the beginning of the first production year. The asterisk notation (B^*) is used to distinguish purchased resources from existing resources (B).

To obtain the solution of this model, three steps are required. First, initial resource conditions, namely the vector $B(0)$, should be analyzed. Second, the quantity of durable inputs purchased in the preceding year should be determined. This can be determined by the computer simulator described in the next subsection. Third, depreciation for durable inputs must be computed. The computing procedure for depreciation is also included in the simulator.

Mono-Period Simulation Model

While the MLP model may provide a basis on *the best possible first move* for the current year, it is very likely that during the course of actual implementation of the plan the manager will follow the *ex ante* plan without any revision. This is quite evident because there are some factors, such as the lumpiness of durable inputs, uncertainty, and psychological inertia, etc., which would prohibit the manager from carrying out the *ex ante* plan thoroughly. One of the purposes of constructing a computer simulation model is to overcome this problem.

The components of the simulation model may be grouped into the several categories. They are shown in a flow chart (Appendix B) and cross-referenced to the chart by numbering categories of the components and blocks of the chart.

Behavioral Constraints

The idea is to take into account the fact that in reality the farmer may not be able to maximize net returns because of uncertainty (in prices and yields), personal preference for keeping an established farming pattern, etc. Thus, the "optimal" farm plan which is based on the assumption of perfect rationality in the *ex ante* planning model may be modified. This feature is considered in the simulation model in Block 5 of Figure 1, Appendix B. In effect, bounds for enterprises in any given year are specified as follows:

$$(1-\alpha)X_{j,t-1} \leq X_{jt} \leq (1+\alpha)X_{j,t-1} \quad (7)$$

where

X_{jt} is the level of enterprise j to be produced in year t ,

$X_{j,t-1}$ is the actual or solution level of enterprise j produced in year $t-1$, $\bar{\alpha}$ and $\underline{\alpha}$ are maximum allowable percentages of increase and decrease, respectively, from the enterprise level in the preceding year.

The upper and lower bounds for enterprise level are entered into the simulator as input data. The programme then checks the "optimal" enterprise levels obtained from the multiperiod model solution with these two limits. The decision rules for determining the exact entering enterprise level are:

- (a) The upper limit is accepted by the simulation model if the "optimal" level is larger than the upper bound of these two limits.
- (b) The lower limit is accepted by the simulation model if the "optimal" level is smaller than the lower bound of these two limits.
- (c) The "optimal" level is accepted if it falls within the limits.

Land Requirements, Cropping Operation

Having adjusted the "optimal" levels of enterprises, the simulation model then determines the acreage of land required to operate these enterprises. This is depicted in blocks 6-8 and the "money" subroutine of the flow chart, Appendix B. If sufficient land does not exist on the farm, purchase and/or a rent of land will be executed. If land is needed to be purchased and sufficient cash is not available, then a long-term loan will be required. Land is assumed to become available for purchase only in units of 20 acres or more.

In flow-chart blocks 9-10 (Appendix B) of the simulator, computations are made for yield and total production for each crop, total acreage of cropland for each crop, cash cost

of crop production, and total requirements of labor and machinery for crop production. Crop yields may be subject to stochastic variation caused by random factors outside farmers' control. All computational procedures except generating stochastic yields are straightforward in nature. Procedures for generating stochastic yields for each crop enterprise are stated in the following mathematical expressions:

$$S_{jt}(Y) = \theta Y_{j,t-1} + (1-\theta)S_{j,t-1}(Y) \quad (8)$$

where

$S_{jt}(Y)$ is the smoothed value (yield) for crop j in time period t ;

$Y_{j,t-1}$ is the actual or generated yield of crop j in time period $t-1$;

θ is the smoothing constant, $0 < \theta < 1$.

This technique is called *exponential smoothing* which is a term applied to a special type of weighted moving average which has been useful in business and economic forecasting [Brown and Meyer, 1961]. It possesses the same nature as that of the expectation model in that the function $S_{jt}(Y)$ is a linear combination of all past observations, and the weight given to any past observation decreases geometrically with its age. However, these two models are not exactly identical. The difference between these two models is that the smoothing constant, θ , is chosen such that sum of squares of the forecast errors is minimum, whereas the coefficient of expectation (β in equation 1) is either estimated from historical data or assumed on the basis of knowledge and judgment.

Since exponential smoothing will lag behind any systematic trend in the data, it is necessary to correct the system for such trend. A simple procedure to handle this problem is to use the differences between successive smoothed values as estimates of

trend. Again, exponential smoothing is used on successive differences to compute an average trend. This procedure may be expressed in the following mathematical forms:

$$C_{jt}(Y) = S_{jt}(Y) - S_{j,t-1}(Y) \quad (9)$$

$$T_{jt}(Y) = \phi T_{j,t-1}(Y) + (1 - \phi)C_{jt} \quad (10)$$

where

$C_{jt}(Y)$ is the change in the smoothed value (yield) for crop j between time period t and the previous time period;

$S_{jt}(Y)$ and $S_{j,t-1}(Y)$ are the smoothed values (yields) for crop j in time periods t and $t-1$ respectively;

$T_{jt}(Y)$ and $T_{j,t-1}(Y)$ are the estimated trend of yield for crop j in time periods t and $t-1$, respectively;

ϕ is the smoothing constant, $0 < \phi < 1$.

After obtaining the smoothed value of yield and the smoothed trend for time period t , the "realized" yield is generated through the simulator by the following procedure:

$$Y_{j,t} = S_{jt}(Y) + T_{jt}(Y) + \epsilon_{jt} \quad (11)$$

where

$Y_{j,t}$ is the actual or generated yield of crop j in time period t ;

$S_{jt}(Y)$ is the smoothed value (yield) for crop j in time period t ;

$T_{jt}(Y)$ is the smoothed trend of yield for crop j in time period t ;

ϵ_{jt} is a stochastic variate affecting the yield of crop j in time period t . Assuming that ϵ_{jt} is normally distributed with zero mean and constant variance, this stochastic variate can be generated by random number generator subroutines [IBM, 1970, p. 77].

THE STUDY FARM

The development of the farm firm growth model was an effort to make *positive* economic analyses of firm growth more realistic and applicable.⁵ To apply the model, much information on farm records and time series data for each business unit is needed. Accordingly, a beef cattle farm cooperating with the University of Kentucky Cooperative Extension Service was selected to provide the unit of analysis for the study.

The farm is a tobacco-beef type located in the Central Bluegrass area of Kentucky. The farm operator is a middle-aged man with good management and farming ability. He has two tenants and offers a 50-50 share of his tobacco production to them. His wife has a part-time job off the farm which returns about \$6,000 per year. At the beginning of 1967, he operated 538 acres of which 402 acres were rented from his father for an annual cash rent payment of \$4,000. The operator's farm records revealed that in 1970 about 67 percent of total cash farm receipts was from calves and feeder cattle. Remaining cash farm receipts came primarily from tobacco production. A summary of the existing farm organization and financial situation for two points in time (January 1967 and December 1970) is shown in Table 2.

As indicated in this table, enterprises on the study farm remained largely unchanged during the four-year period, 1967-70. This table also reveals that the operator seems to have a strong preference for the beef enterprise--not atypical for Central Kentucky farmers. Yet, the beef herd did not show significant expansion during this period--also not atypical for many Central Kentucky farms. A fall calving system was used even though it usually requires

⁵A more complete description of the model is given in the Appendixes.

TABLE 2

FARM ENTERPRISE ORGANIZATION AND FINANCIAL SITUATION OF THE CASE FARM
 JANUARY 1967 AND DECEMBER 1970

Item	Unit	January 1967	December 1970
Enterprise Organization			
Crops:			
Burley tobacco	Acres	8.68	8.62
Corn	Acres	0	22
Corn silage	Acres	10	25
Hay	Acres	33	17
Pasture I	Acres	249.32	271.38
Pasture II	Acres	184	184
Livestock:			
Cow-calf	Head	71	68
Feeder cattle	Head	62	75
Financial Situation			
Assets:			
Cash	Doll.	9,544	2,100
Bonds & other securities	Doll.	24,500	15,000
Livestock on hand	Doll.	24,200	43,000
Crops on hand	Doll.	0	5,000
Machinery	Doll.	6,900	27,500
Buildings	Doll.	4,000	---
Land	Doll.	85,000	95,000 ^a
Total	Doll.	154,144	185,600
Liabilities:			
Long-term debt	Doll.	40,000	37,500
Intermediate-term debt	Doll.	14,000	34,000
Short-term debt	Doll.	0	10,000
Total	Doll.	54,000	81,500
Net Worth		100,144	104,100

^aIncluding the value of buildings.

somewhat higher production costs. Interviews revealed, however, that the operator intended to soon adopt a spring calving program. It seems reasonable to expect that if the current trend of increasing demand for beef continues, the operator's beef cow herd is likely to expand in the years to come.

In contrast to a rather constant beef herd size, this farm has, in terms of the financial information, exhibited growth from 1966 to 1970. This is shown in the lower portion of Table 2. Owing to an increase in feeder cattle, land value appreciation, and the replacement of farm machinery, total assets increased by \$32,456 during the period. However, increases in assets were largely offset by substantial increases in liabilities, resulting in a very slight increase in net worth (\$3,956).

Because of the complexity of the actual environment of farming, certain general assumptions are needed to facilitate the application of the model to the farm. These are outlined and briefly discussed:

- (1) The farm operator is interested in expansion of his farm business, subject to the maximization of net returns to his labor and management, but also subject to certain withdrawals of cash from the net income stream to be used for family consumption. Maximization of net returns is consistent with providing a large amount of internal capital for the purpose of reinvestment. On the other hand, the firm and the household always face a conflict over how income flows should be allocated, i.e., allocation for both farm family consumption and reinvestment in the farm business as a basis for later income and consumption [Heady, 1965, p. 423].
- (2) The relationship between the operator and his tenants remains unchanged during the course of the study. In other words, the farm operator is assumed to

stay with his decision that he will offer a 50 percent share of gross proceeds from tobacco production to his tenants primarily in return for labor services to produce the tobacco. Based on conversations with the operator, it is felt that this is his way of assuring him sufficient labor supply for the tobacco production.

- (3) The cash rental arrangement between the operator and the landlord (operator's father) is also assumed to remain effective during the period of the study.
- (4) Since a review of the farm records revealed a stable enterprise organization during the 4-year period 1967-70, it is assumed that the same combination of enterprises will be continued during the next 4 years. This assumption indicates that the operator's preference for existing enterprise remains the same.

ANALYSIS OF RESULTS

To facilitate presentation, this section is divided into four subsections. First, the problem of sample size is discussed. This is followed by an examination of the model validity, including historical comparisons of simulated and actual values of key variables. Third, experimentation with the model is illustrated. Finally, actual experimental results are presented and analyzed.

Sample Size

Like most computer simulation models, the growth model employed in this study is intended to yield information about the average level of certain response variables, such as average values of assets and net worth. As estimates of population averages, the sample averages computed from several runs of the model on a computer will be subject to

random variation and likely will not be exactly equal to the population values. In short, these computer-generated sample averages are random variables because they are a function of a number of other variables, several of which are stochastic in nature. The larger the sample (i.e., the more runs one observes) the greater the probability that the sample averages will be "close" to the population values. With computer simulation, one method of increasing sample size is to increase the total length of a simulation run; namely, the simulated time. Alternatively, the sample size may also be increased by replicating simulation runs (replications) within a given length of run. Replications can be achieved by using different sets of random numbers generated from the computer. Since the current model attempts to trace out the time path of the farm firm growth process using a representative beef cattle farm for a span of several years, the replication method was employed.

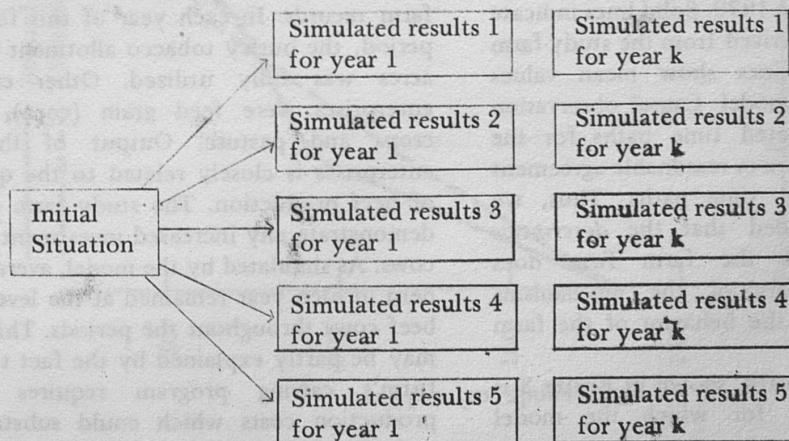
In replicating the experiment with a simulation model, an important constraint is computer time (cost). Accordingly, given a benchmark run of the model, five replications were executed to obtain a sample for each year. This is equivalent to saying that five exactly homogeneous experimental units (i.e., five duplications of the study farm) were

prepared for the computer run of the model. Therefore, a sample of each response variable can be computed from the simulated data and an average time path of the response variables may be traced out over a specific time period. This operational hierarchy is sketched below.

Validation of the Model

One of the most elusive methodological problems associated with computer simulation is validating the model. There are a few propositions in the literature concerning this subject [Naylor, 1968, pp. 310-320]. In general, however, two criteria seem appropriate in validating simulation models. First, how well do simulated values of the response variables compare with known historical data, if historical data are available? Second, how capable is the simulation model in predicting the behavior of the real system in future time period(s)?

This study majored on developing a computerized growth model of the farm firm for *descriptive-predictive* analysis, i.e., construct a computerized behavioral mechanism describing and predicting the ways in which farm operators will perform over time. Accordingly, the model so developed



must "reasonably" represent actual farming behavior, so that it may then be used to conduct alternative practices and ways of farming, i.e., experimental prediction. Because of this nature of the model, it is contended that the procedure of historical validation suffices. In comparing the time paths of selected response variables with the historical data, one should not expect the model to be an exact replication of the real world. However, the model should be able, if it is to be of any relevance, to generate "reasonable" time paths of general behavior of a farm firm or similar types of farm firms.

Total assets, net worth, and the number of beef cows were used as the key response variables for validating the model. The decision to select these three response variables was based on two factors. First, while there are dozens of response variables in the system of the growth model, total assets and net worth are deemed to be most important for a financial-growth analysis. Second, since the model was applied to a tobacco-beef farm, the growth of the farm, if any, would likely result from the expansion of the beef cattle herd.

Graphical comparisons of the simulated time paths and the observed values of the selected three response variables of the model are presented in Figures 2, 3, and 4. Comparisons are for a 4-year period—beginning in December 1966 and ending in December 1970. Solid lines indicate the actual values derived from the study-farm records. Broken lines show mean values generated by the model. Causal observation reveals that simulated time paths for the response variables are in reasonable agreement with the observed time paths. Thus, we tentatively concluded that the *descriptive* growth model of the farm firm does encompass essentials of the mechanisms which determined the behavior of the farm firm operations.

Annual net worth, shown in Figure 3 is the only variable for which the model

produces inaccurate results. The model generates a time path for annual net worth that is uniformly above the actuals. Yet, from 1967 to 1970 the slope (rate of increase) of the simulated results are very close to the actual data. Initial departure in magnitude of simulated results from actual observations for this response variable stems from difficulty in incorporating into the model the precise debt payment schedule for the study farm. Specifically, the study farm operator, because of personal relations with lending agents, could pay back debt principal on a very slow or liberal schedule. An examination of the farm records reveals that the operator paid back debt principals with uneven payment amounts over time. He paid more in some years and less or even none in certain other years, depending upon the flow of net income and personal factors. This kind of phenomenon occurs very often in the real world and is an important fact which should be considered in any study of farm firm growth. Unfortunately, it seems not easy to describe this sort of personal behavior in an abstract model because of the non-systematic nature of such behavior.

Simulated enterprise organizations for the comparative period, 1967 through 1970, are presented in Table 3. Note that most enterprise levels underwent little change during this period. These simulated results are consistent with actual data as indicated by the farm records. In each year of this four-year period, the burley tobacco allotment of 9.57 acres was fully utilized. Other cropping enterprises were feed grain (corn), forage crops and pasture. Output of the feed enterprises is closely related to the quantity of beef production. The study farm did not demonstrate any increased investment in beef cows. As simulated by the model, average cow herd in each year remained at the level of 71 beef cows throughout the periods. This result may be partly explained by the fact that the farm's calving program requires higher production costs which could substantially

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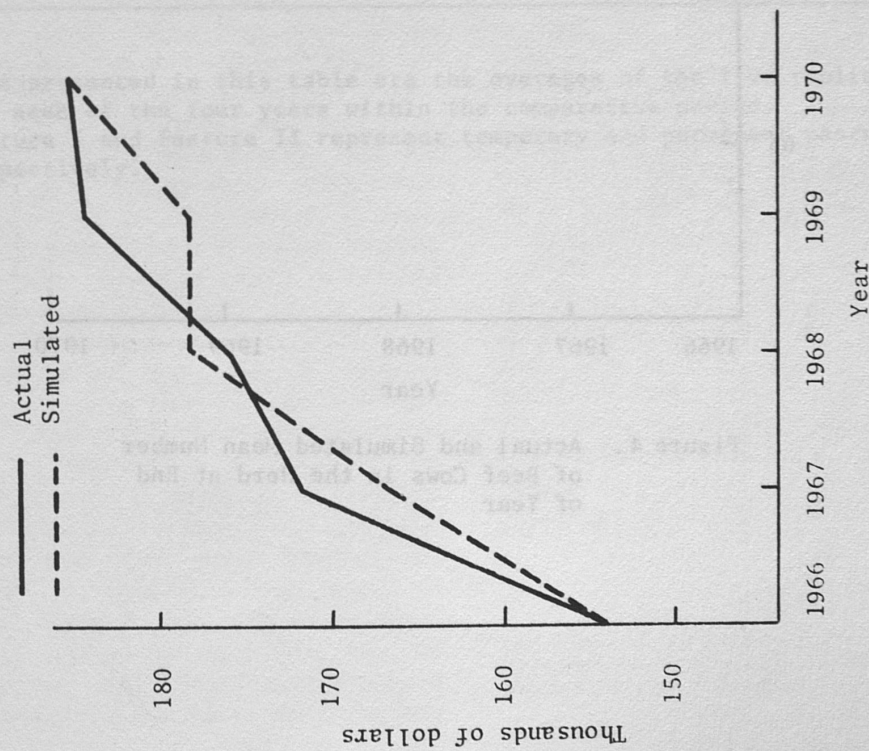


Figure 2. Actual and Simulated Mean Values of Total Assets at End of Year

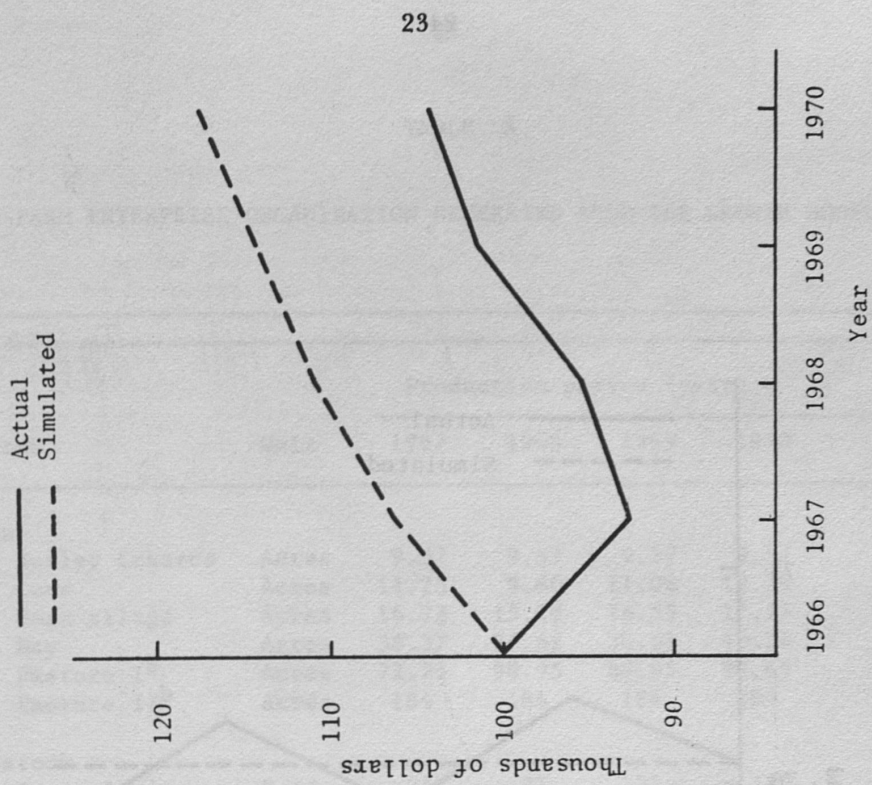


Figure 3. Actual and Simulated Mean Values of Net Worth at End of Year

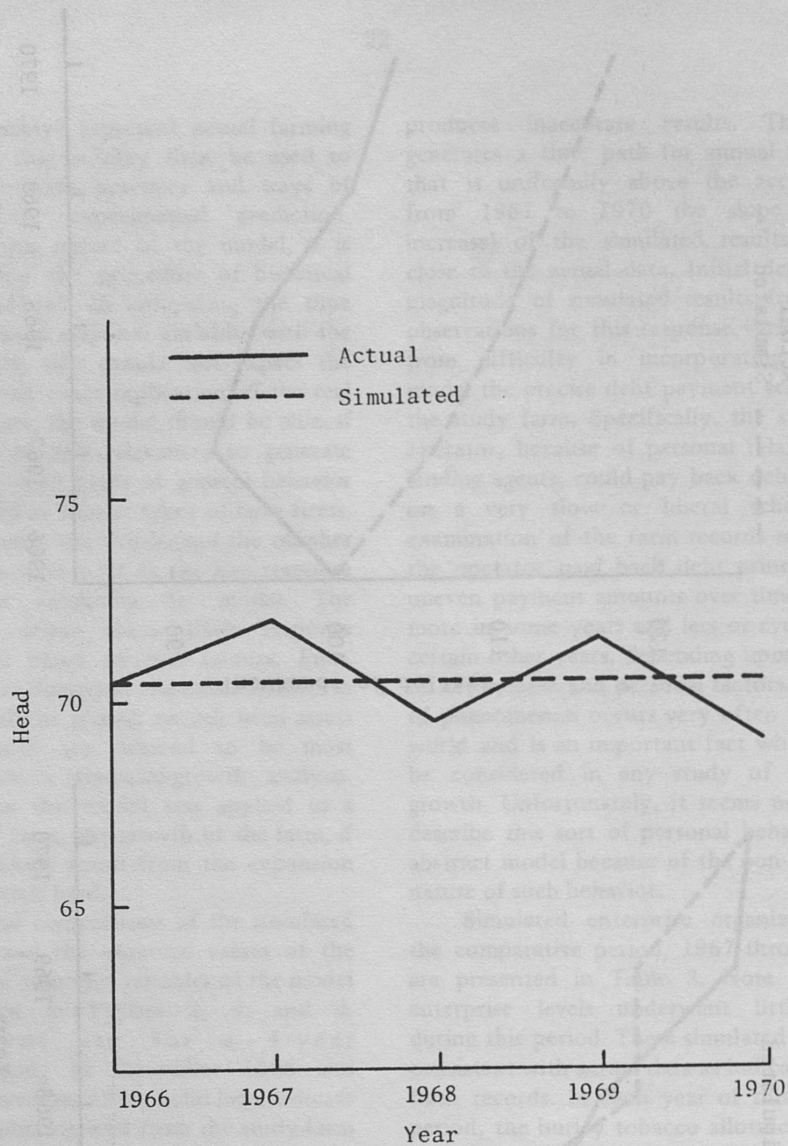


Figure 4. Actual and Simulated Mean Number of Beef Cows in the Herd at End of Year

TABLE 3

FARM ENTERPRISE ORGANIZATION GENERATED FROM THE GROWTH MODEL^a

Item	Unit	Production period (year)			
		1967	1968	1969	1970
Crops					
Burley tobacco	Acres	9.57	9.57	9.57	9.57
Corn	Acres	11.25	9.80	11.06	12.29
Corn silage	Acres	16.73	15.18	16.55	17.75
Hay	Acres	38.37	35.62	38.01	40.34
Pasture I ^b	Acres	72.75	90.95	84.95	98.65
Pasture II ^b	Acres	184	184	184	184
Livestock					
Cow-calf	Head	71	71	71	71
Feeder cattle	Head	62	109	88	106

^aData presented in this table are the averages of the five replications for each of the four years within the comparative period.

^bPasture I and Pasture II represent temporary and permanent pastures, respectively.

pull down net returns even though feeder calf prices have been reasonably high in recent years. Consequently, nonprofitability, together with risk in such a livestock operation provides the farm operator with little incentive to expand the cow herd.

In contrast to the stationary beef cow herd, the feeder cattle enterprise expanded sharply during the 4-year period--consistent, again, with actual data. Under this system of cattle feeding, feeder calves typically are purchased in one year (often in the fall) and sold the following year (frequently to farmers in other states) for further feeding to slaughter or heavier weights. Since feeder cattle are kept on farms for only one year or less, this type of beef enterprise involves much less production risk than the cow-calf operation. However, the farm operator must be more responsive to the changing market situation. Favorable cattle prices in recent years apparently, then, have been one of the primary reasons for the expansion of study farm's feeder cattle enterprise. This expansion results in, as shown in Table 4, a slight increase in land use for forage crops and temporary pasture (pasture I).

Financial and other information arising from the model over the 4-year comparative period is shown in Table 4. Because of the complexity of the bond and security market, values of bonds and securities in each year were assumed to be equal to \$17,682--the 4-year average computed from the study farm records. The increased value of total assets was partly due to expansion of the feeder cattle enterprise, added investment on machinery and appreciation of land values.⁶ The increase of total assets was, though, largely offset by an increase in total liabilities,

⁶Based on the operator's judgment as revealed from the farm records, and Kentucky Agricultural Statistics, a value of 3.1 percent was assumed for the yearly appreciation of land values.

resulting in only a slight increase in net worth during the period.

The large debt incurred in the beginning year (1966) could be one of the factors contributing to the lack of net worth growth over the 4-year period. Investment capital outlay in this period was confined entirely to the purchase of machinery and equipment. Yearly investment ranged from \$2,595 to \$3,652. Seventy-five percent of the yearly investment was simulated to be financed by intermediate-term loans. Since neither land nor buildings was purchased after the beginning year, no long-term loans were employed in any year within the period. The magnitude of short-term loans tended to increase over the 4-year period. Hence, the farm operator with insufficient internally generated capital must rely on short-term borrowing to finance seasonal expenses. The operator also is likely to employ short-term loans to refinance debts contracted previously. This kind of financial arrangement can be easily made by any farmer who, like the study farm operator, has good relations with lending agents. As a tide of prosperity or depression brings about more or less income, the farm operator may refinance his debts, consolidating them at one time, reducing them at another time, or borrowing from different sources in order to expand the farm business or simply to remain solvent. Note (Table 4) that disposable income increases after 1967. This improvement in income occurred as the feeder cattle enterprise expanded and interest payments on the outstanding long-term debt declined.

Variability of assets and net worth simulated by the model is analyzed in Table 5. Average values of total assets and net worth are included along with their standard deviations and coefficients of variation. Variation in these two response variables becomes greater as the number of years projected lengthened. This phenomenon probably is due to the recursive nature of the

TABLE 4

SUMMARY OF SIMULATED FINANCIAL OUTCOMES FOR THE CASE FARM^a

Item	December 31, Year				
	1966	1967	1968	1969	1970
Assets					
Cash	9,544	3,564	14,817	8,794	9,620
Bonds & securities	24,500	17,682	17,682	17,682	17,682
Crops on hand	0	7,210	6,626	7,240	7,647
Livestock on hand	24,200	35,106	32,421	34,846	37,544
Machinery	6,900	12,464	12,772	13,436	14,203
Buildings	4,000	3,733	3,466	3,199	2,932
Land	85,000	87,635	90,352	93,153	96,041
Total	154,144	167,394	178,136	178,350	185,669
Liabilities					
Long-term	40,000	38,667	37,333	36,000	34,667
Intermediate-term	14,000	18,230	16,543	17,251	18,012
Short-term	0	3,928	12,972	10,591	15,199
Total	54,000	60,825	66,848	63,842	67,878
Net Worth	100,144	106,569	111,288	114,508	117,791
Capital Investment^b					
Land		0	0	0	0
Buildings		0	0	0	0
Beef cows		0	0	0	0
Machinery		3,652	2,595	2,779	3,167
Disposable Income, (after taxes) ^b		6,000	12,620	8,410	9,172

^aValues, except those in the column under 1966, are the averages of the five replications for each of the four years within the comparative period.

^bDuring years 1967, 1968, 1969, and 1970, respectively.

TABLE 5
 MEANS AND VARIATIONS OF TOTAL ASSETS AND NET WORTH
 AS SIMULATED BY THE GROWTH MODEL

Year	Assets			Net Worth		
	Mean	Standard Deviation	C.V. ^a	Mean	Standard Deviation	C.V. ^a
1967	167,394	1,526	0.9	106,569	1,596	1.5
1968	178,136	3,006	1.7	111,288	3,229	2.9
1969	178,350	10,453	5.8	114,508	5,530	4.8
1970	185,669	18,162	9.8	117,791	8,402	7.1

^aCoefficients of variation are expressed in percentages, i.e., the percentage that the mean is of the standard deviation.

model. Effects of "extreme" values associated with the stochastic variables accumulate from year to year over time through the lagged endogenous variables--consistent with the fact that as the farm firm planner looks further into the future he is faced with more risk and uncertainty, and hence, greater variations in the future outcomes. Thus, one could conclude that the growth model of the firm developed in this report is more applicable for an intermediate-term analysis. If this model must be applied to a long-term study, the number of replications (i.e., the computer runs) should be increased in order to obtain "better" estimates (more accurate and precise) of the mean values for response variables.

Experimentation with the Model

Like most computer simulation models, the multiperiod programming-simulation farm firm growth model employed in this study permits experimentation--that is, computer simulated experiments about the real farming process. One of the merits of such a computer model is to reduce the time and cost involved, as compared to real-world, controlled experiments. Once the validity of the model is accepted, it may be used to conduct experiments of various sorts.

Essentially, experimentation with the model allows the researcher to make certain changes in the model and then observe the effects of these changes upon the endogenous variables, e.g., total assets. Experimentation may be undertaken with the intent of evaluating alternate business or organizational structures, determining effects of parameter changes on managerial behavior, or testing different decision rules in the model. Because of limited time and computing cost for this study, only a simple simulation experiment was conducted. However, this should demonstrate the possibility of using

computer-based experiments to analyze a variety of firm growth problems. Since the study unit is a tobacco-beef farm, the primary purpose of this experiment is to examine the effects of alternative production systems pertaining to beef enterprises upon terminal states of the farm firm growth or survival. A 4-year period (1971 through 1974) was chosen for analysis. The selection of this period--intermediate in length--is based on the analysis of preliminary results. Specific alternative structures to be considered in the experiment are listed as follows:

- (1) Alternative 1--*Continuing the existing beef cattle production systems.*

The purpose of including this alternative is to provide a base on which a *comparative analysis* may be made. Thus, this alternative may be regarded as a "control treatment". As may be recalled, the existing beef cattle systems operated by the study farm are a cow-calf (fall calving) program and a feeder cattle enterprise.

- (2) Alternative 2--*Changing to a spring calving program.*

This treatment substitutes spring calving program for the fall calving program. In essence, it adopts a recommendation from beef cattle specialists that spring calving is a "better" time for calving, because during the spring months it is easier to handle the cows and calves and costs less to maintain them. Under a spring-calving program the cow's highest nutritive requirement during her lactation and before rebreeding coincides with the natural grass cycle, explaining that the ample forage will meet most of herd needs. Therefore, one should anticipate a favorable effect of this program on farm business or growth variables.

- (3) Alternative 3--*Changing the variability of market prices, spring calving program.*

Under this treatment, the variability of respective prices for calves and feeder cattle was arbitrarily lowered by 50 percent, i.e., decrease by 50 percent respective standard deviations for the random variable associated with prices. In practice, a lower variability of prices may be achieved through the improvement in livestock marketing strategies (such as hedging). A smaller degree of variability in the realized price of beef cattle would appear to decrease the complexity of decision-making. As a result, growth patterns of the farm firm, in terms of endogenous variables, are expected to be different from the other two alternatives.

Experimental Results

The experiment consisted of three runs, one for each of the three alternative systems described above. In each run the farm situation was simulated for a period of 4 years (i.e., 1971 through 1974), with five replications for each alternative. The main endogenous variables considered in the analysis were total assets, net worth, net farm income, and the number of beef cows in the herd.

Since meaningful farm firm growth usually results from expansion of farm production, it is useful to examine projected farm enterprise organizations under the three alternatives. These results are presented in Table 6. The projected results indicate that the cow-calf enterprise could grow some during the projected period. Moreover, the feeder cattle enterprise has a higher level than that in the base period, 1967-70. Such expansion stems primarily from a reduction in interest expenses and principal payments for the old debts, releasing operating capital for the feeder cattle activity. Note that under alternatives 2 and 3 the tobacco allotment is,

with exception of 1971, not fully used in this projected period. An explanation for this result is that as beef production expands, more of the limited operating capital would be allocated to feed production, hence, causing a contraction of tobacco production.

The substitution of spring calving program for the fall calving operation (Alternative 2) results in an expansion in both the beef cow herd size and the feeder cattle enterprise. As shown in Table 6, the number of beef cows in the herd is increased to 93 cows at the end of 1974; the feeder cattle enterprise expands to 152. The expansion in beef production under this alternative is mainly due to the relatively low production costs of the spring calving operation.

The projected enterprise organization under the third alternative (spring calving program and less variability in prices for calves and feeder cattle, respectively) exhibits the same pattern as that of alternative 2. However, the expansion in beef cow herd size tends to be more rapid than that under alternative 2. Perhaps the lower price variability would ease the complexity of the decision-making process, thus encouraging the operator to expand his farm business whenever markets indicate favorable opportunities.

In general, total values of assets under all alternatives, except alternative 3, tend to increase slightly during early years of the projected period (Figure 5). Still, with no land-value appreciation the value of total assets would be decreasing during these years. This is due to the fact that existing assets are depreciated out while no new assets are added.

Projected average patterns or time paths for net worth under the three alternatives, as shown in Figure 6, differ only slightly from those for assets. Because of the decreasing liabilities and the benefits from the appreciation of land values, the study farm was simulated to have increasing values of net

TABLE 6

COMPARISON OF SIMULATED FARM ENTERPRISE ORGANIZATIONS,
THREE ALTERNATIVE PRODUCTION SYSTEMS ON THE STUDY FARM,
1971 THROUGH 1974^a

Item	Unit	1971	1972	1973	1974
Alternative 1, Present System					
Crops					
Burley tobacco	Acres	9.57	9.57	9.46	9.42
Corn	Acres	11.91	10.93	11.65	13.71
Corn silage	Acres	17.32	16.41	16.96	18.47
Hay	Acres	39.61	37.75	38.65	41.07
Pasture I	Acres	106.25	100	94	103.35
Pasture II	Acres	184	184	184	184
Livestock					
Cow-calf	Head	71	71	71	71
Feeder cattle	Head	124	119	104	116
Alternative 2, Spring Calving Program					
Crops					
Burley tobacco	Acres	9.57	9.53	9.54	9.23
Corn	Acres	8.26	9.30	10.41	8.58
Corn silage	Acres	8.07	9.08	10.16	8.37
Hay	Acres	15.69	17.66	19.76	16.29
Pasture I	Acres	112.45	123.35	141.70	151.90
Pasture II	Acres	184	184	184	184
Livestock					
Cow-calf	Head	77	81	88	93
Feeder cattle	Head	124	123	136	152
Alternative 3, Less Price Variability					
Crops					
Burley tobacco	Acres	9.57	9.35	9.55	9.39
Corn	Acres	8.26	6.55	7.48	6.91
Corn silage	Acres	8.07	6.40	7.31	6.75
Hay	Acres	15.69	12.45	14.21	13.13
Pasture I	Acres	112.45	113.80	115.10	129.40
Pasture II	Acres	184	184	184	184
Livestock					
Cow-calf	Head	77	83	90	96
Feeder cattle	Head	124	121	96	110

^aData presented in this table are the averages of the five replications for each of the four years within the projected period.

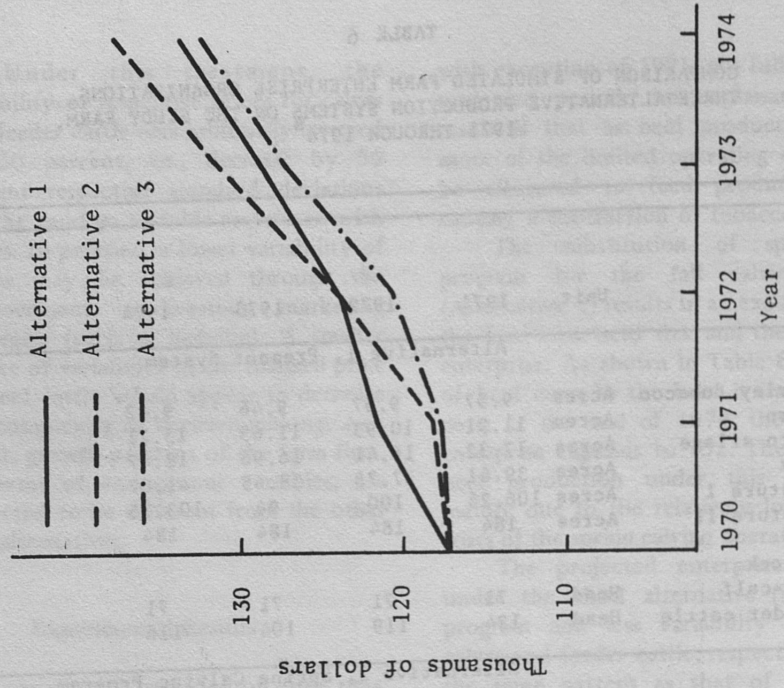


Figure 6. The Projected Time Paths for Net Worth under the Three Operating Alternatives

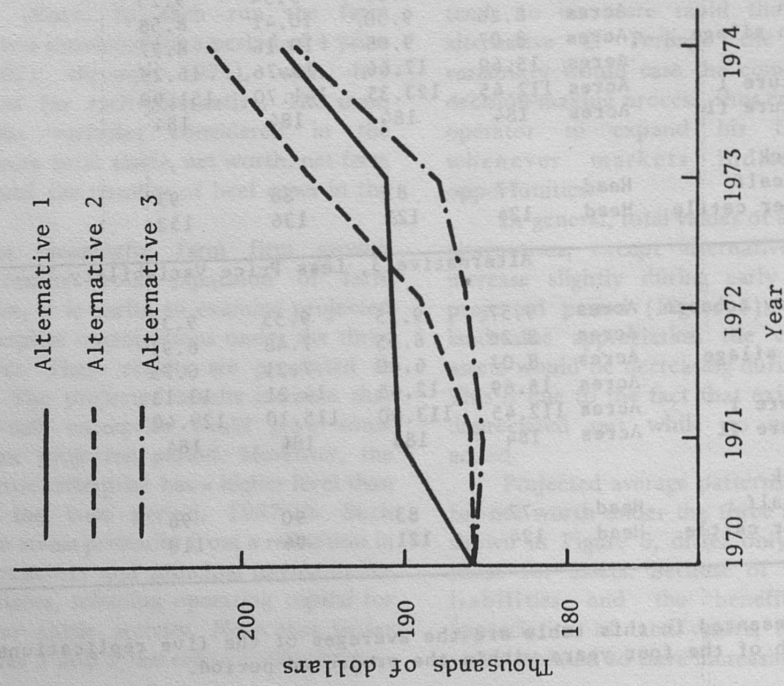


Figure 5. The Projected Time Paths for Total Assets under the Three Operating Alternatives

worth during the projected period, although the comparative simulated results did not show significant differences in capital investment.

In addition to the analysis of model-generated data presented in the preceding paragraphs, it is important and meaningful to examine terminal states of the farm business under the three different production or operating alternatives. The analysis of variance was employed for such data analysis. In terms of experimental design terminology, the three operating alternatives are *factors* since they were purposefully selected for the experiment whereas the assets and net worth are *responses* because they are endogenous (output) variables in the experiment.

Output data (replications) generated by the model are tabulated in Tables 7 and 8. For each alternative, the sample mean and standard deviation were calculated. In conducting the analysis of variance the output data were transformed into logarithms because this transformation has two advantages [see H. Sheffe, 1959, pp. 83-87]. First, the transformed random variable is more nearly normally distributed than the original variable. Second, the variance of the transformed variable tends to be constant over the alternative treatments. Computed statistics for these analyses are presented in Tables 9 and 10.

From these two tables one can observe that the calculated F values for both response variables (assets and net worth) are less than one, indicating that the data generated by the computer experiments supports the null hypotheses that expected terminal states of the farm firm, in terms of assets and net worth values, at the end of 1974, are the same for each of the operating alternatives.

These statistical findings do not depart from expected reality. As stated earlier, under the spring-calving system, hay and pasture are

the major sources of energy. But, of the two sources, pasture is the primary feed. Consequently, this cow-calf production system involves less machinery requirements for feed production. Therefore, significant firm growth in terms of assets or net worth would primarily stem from a substantial expansion in the cow herd. This is not likely to occur during a short period of 4 years. Over a period of years, if a spring-calving program is adopted, one would expect a significant increase in growth.

As expected, the spring-calving operation generates higher net farm income than the fall-calving program. Undoubtedly, this would provide the operator with greater incentive and more internal funds to expand his beef production.

A final caveat should be made before concluding this section. The reader may have noted that the crop acreages in Tables 4 and 6, if totaled for any year or production system, are considerably less than the actual total acreage existing on the study farm in years 1967-1970. In other words, the model generated unused acreage. This is indicative of the fact that capital was more constraining than land. One view is that this acreage is strictly idle, i.e., it has a zero opportunity cost. Another view is that the acreage does have some positive opportunity cost. For example, this land might be leased out for pasture purposes. If so, then, revenues generated from such returns obviously would increase the income stream in the years for which simulations were made, and such revenues possibly be used to decrease liabilities--therefore, increasing simulated net worth values. The former view, however, seems more plausible for purposes of this study, since most farmers in the geographic area of the study farm tend to have an excess of land relative to working capital. Therefore, the unused acreage on the study farm is more appropriately construed as idle land.

TABLE 7

OUTPUT DATA GENERATED BY THE GROWTH MODEL
FOR NET WORTH AT THE END OF 1974

Replication	Alternative 1	Alternative 2	Alternative 3
1	145,591	138,628	137,175
2	119,569	126,628	129,267
3	117,361	119,038	120,346
4	167,878	166,974	165,941
5	<u>119,455</u>	<u>140,078</u>	<u>110,142</u>
Mean	133,971	138,269	132,574
Standard Deviation	22,242	18,253	21,202

TABLE 8

OUTPUT DATA GENERATED BY THE GROWTH MODEL
FOR TOTAL ASSETS AT THE END OF 1974

Replication	Alternative 1	Alternative 2	Alternative 3
1	194,642	189,915	188,497
2	182,843	182,876	189,824
3	170,857	178,737	177,354
4	245,873	232,754	239,485
5	<u>195,229</u>	<u>225,228</u>	<u>189,693</u>
Mean	197,889	201,902	196,970
Standard Deviation	28,623	25,190	24,332

TABLE 9

STATISTICS FOR ANALYSIS OF VARIANCE FOR
TOTAL ASSETS AT THE END OF 1974

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	
Between Alternatives	0.0000007	2	0.00000035	F<1
Error	0.0329583	12	0.00274653	
Total	0.0329590	14		

TABLE 10

STATISTICS FOR ANALYSIS OF VARIANCE FOR
NET WORTH AT THE END OF 1974

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	
Between Alternatives	0.0008850	2	0.00044250	F<1
Error	0.0502930	12	0.00411733	
Total	0.0502930	14		

SUMMARY AND CONCLUSIONS

This report has demonstrated the potentiality of the multiperiod linear programming-simulation model in describing and predicting growth patterns of the farm firm. Although the conclusion was drawn from an application of the model to an individual farm, it is felt that the model should be equally, if not more, appropriate for studying a "group" of the farm firms in an area. Information regarding growth process of a group of farm firms may be obtained from the model if (1) decision behavior can be generalized and input-output relations can be synthesized properly for a group of the farm firms, (2) the aggregate aspects of the growth of farms are taken into consideration. That is, any competition among the farm firms for resources available for growth should be taken into account.

In operating the model for the empirical study, the IBM, Mathematical Programming System/360 (MPS/360) routine was used to solve the multiperiod linear programs while the mono-period simulation model of the farm operation was programmed with FORTRAN. Since the MPS routine is a "built-in" program, it seems difficult to link this routine with the simulation program—thus, the flow of input data between these two programs was accomplished externally ("by hand"). Further efforts to solve this technical problem would substantially reduce time needed to operate the model.

One of the objectives of the study was to demonstrate the applicability of the multiperiod linear programming-simulation model to predict farm firm growth variables. To meet this objective a beef cattle farm located in central Kentucky was selected to provide the unit of analysis. Using this model, expected changes in growth patterns of the study farm with respect to the selected variables (price variability and management

strategies) were predicted. The effects of these selected variables on the expected terminal states of the farm business were also analyzed.

Validity of the model was tested by comparing model-generated time paths of total assets, net worth, and the number of beef cows with historical data. Graphical comparisons of the simulated results and the observed values for these three response variables reveal that simulated time paths (1967 through 1970) are in reasonable agreement with the observed time paths. Thus, one can conclude that the model is *descriptive* and that it does encompass some of the mechanisms which determined the behavior of the farm firm. As a complement to the testing of model validity, the model-generated farm organization and enterprise levels during the 4-year period (1967 through 1970) were also compared with actual data. In general, the simulated data were consistent with actual happenings as indicated by the farm records.

To predict expected change in growth patterns of the study farm with respect to price variability and alternative management strategies, simple (but typical) computer-based experiments were conducted. Experimental results indicate that if the fall calving practice continues the cow herd size will remain the same during the projected period (1971 through 1974). Substitution of a spring calving program for the fall calving operation resulted in an expansion in both the beef cow herd size and the feeder cattle enterprise. This is mainly due to relatively low production costs of the spring calving operation.

The predicted beef cow herd size under alternative 3 (spring calving and lower market price variability) tends to expand more rapidly than that under alternative 2 (spring calving alone vs. fall calving as alternative 1). Obviously, less price uncertainty would reduce the complexity of the decision-making

process--thus encouraging the farm operator to expand his farm business whenever markets indicate favorable opportunities.

Expected terminal states of the farm business (in terms of total assets and net worth values) at the end of 1974 were not statistically different under the three management alternatives. These findings should be interpreted in the light that, under the spring-calving system, hay and pasture are the major sources of energy. But, of the two sources, pasture is the primary feed. This implies that the spring calving system requires less machinery investment for feed production. Therefore, any significant firm

growth in terms of assets or net worth would primarily stem from a substantial expansion in the cow herd. This is not, however, likely to occur during a short period of four years.

As was expected, projected patterns for net farm income under the three alternatives showed that spring calving operation would generate higher net farm income than the fall calving system. The results also indicated that alternative 2 tends to generate a higher but more variable net farm income than alternative 3. This finding implies that alternative 3 may be the most preferable one among the three since it would lessen uncertainty in farm planning.

growth in terms of both in real world
 generally seen from a substantial expansion
 in the last half of the century, likely
 to occur during a short period of four years
 at least. The period of 1970-1975
 and from 1980-1985 under the same conditions
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project that following the same operation
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APPENDIX A

MULTIPERIOD LINEAR PROGRAMMING MODEL

It is assumed that at the beginning of each production year the farm operator makes a 5-year farm plan. In other words, the multiperiod linear programming (MLP) model includes a horizon of five years. For brevity, only a 2-year version of the complete model is shown in Table 1 of this appendix. An overview of the tableau in this table reveals that it is indeed a species of the ordinary linear programming framework. This example model is divided into two planning periods (i.e., two years). The symbols, $1P_j$ and $2P_j$ ($j=1, 2, \dots$) appearing on the upper part of the tableau represent activity j in production years one and two, respectively. Likewise, $1R_i$ and $2R_i$ ($i=1, 2, \dots$) denote the constraints i in year one and year two, respectively.

The Activities

Activities included in a particular year, say year one, are grouped into six sectors: Production, labor hiring, investment, financial, accounting and transfer.

The Production Sector (P_1, \dots, P_{10})

This sector consists of all "regular" cropping and livestock activities. Included in the cropping program are those that were actually grown during the 1967-70 period, viz., burley tobacco, corn, corn silage, hay and pasture. A buying activity for corn was also included. The input-output coefficients indicating annual resource requirements for each activity were derived from farm records and various farm planning manuals [Allen and Browning, 1971; Allgood *et al.*, 1971, 1971a; Ray and Hudson, 1969]. The symbols Y_C and Y_H are "expected" yields of corn and hay, respectively. These expected yields were calculated from the expectation model each year when the model was "programmed." Specific computational procedures were discussed by Chien [1972].

Livestock activities include the beef cow-calf program and feeder cattle enterprises. Since the credibility of the proposed model was tested by comparing generated results with the actual outcomes, during 1967-70 the fall calving program was considered to be the only alternative for the cow-calf activity. A spring calving program was included in the experimental analysis (see the later section of this report). A 95 percent calf crop and a 10 percent replacement rate for cull cows were assumed. Replacement of cull cows was assumed to be accomplished from internal retention of heifer calves. Feeder calves were assumed to be sold at 470 pounds.

Following the operator's practice, young stock was assumed to be purchased in October and sold the next October as a stocker. Accordingly, two activities, young stock buying and stocker selling, were included in the model. Purchase and sale weights were assumed to be 450 pounds and 800 pounds, respectively. Feed requirements were estimated from the farmer's records.

Other coefficients were obtained from the same sources as those mentioned in the cropping activities. The symbols, CR_{ca} and CR_{fc} denote "expected" cash receipts from feeder calves and feeder cattle, respectively. The notation CC_{ys} in the young stock buying activity is the coefficient indicating "expected" cash cost of young stock. Computational procedures for these expected values are similar to those for computing expected crop yields.

The Labor Hiring Sector (P11, . . . , P18)

Since the farm operator could hire his tenants for farm work (in addition to tobacco production) for a wage rate of \$0.75 per hour, which was well below the prevailing farm wage rate, labor hiring activities were grouped into two categories. Each category contains four separate activities in accordance with the four calendar quarters. A wage rate equal to \$1.50 per hour for farm labor other than tenants was assumed.

Investment Activities (P19, . . . , P23)

This sector contains five activities allowing for purchase of real estate (land and buildings) accompanied by required additions of machinery and livestock (beef cows) investment. Investment activities for land and livestock are expressed in physical units while the purchase activity for machinery is treated in dollar value units. This is to avoid problems in specification and computation for the MLP model, because a great deal of machinery with different ages and types may be included in the model. In practice, this consideration also can be justified on the grounds that a detailed specification of machinery is very unlikely to come into the operator's mind when he is making a long-run prospective plan. The same procedure is also followed for investment in new buildings. Land was assumed to become available for purchase in modules of 20 acres. The price of land per acre was assumed to be \$500 and \$180 for tillable and nontillable land, respectively.

Since machinery and buildings are depreciable assets, one dollar's worth of additional investment provides less than one dollar's worth of assets in years subsequent to purchase. The actual magnitude depends on the production life of the asset. In the MLP model, respective lives for machinery and buildings were assumed to be 10 and 30 years, respectively.

The purpose of including a cow-buying activity (P23) is to provide for operation expansion, if profitability so indicates. A breeding cow was assumed to have a 10-year productive life, and purchase price was assumed to be \$250 per cow.

All of the previously discussed investment activities are made possible through secured loans. Even though any investment in any production period will reduce capital borrowing capacity, it also strengthens the borrowing capacity for the subsequent periods. Detailed discussion on the financial activities is provided in the following sector.

The Financial Sector (24, . . . , P27)

Four capital borrowing activities are included in the financial sector: long-term borrowing, intermediate-term borrowing for machinery, intermediate-term borrowing for cows, and short-term borrowing.

The long-term borrowing activity is to provide funds for the purchase of land and farm buildings. Maximum borrowing capacity for this type of loan was assumed to be 60 percent of total farm assets minus total outstanding debts. Thirty equal annual installments for the repayment of principal were assumed. Payments on principal were assumed to be paid at the end of each year, starting from the year when the loan was initiated. Thus, borrowing capacities in succeeding years are not mutually exclusive of the borrowing decision in a specific year. For example, a \$1 loan in year 1 requires not only \$1 borrowing capacity in that year but certain borrowing ability in all subsequent years (\$0.967 in year 2, \$0.934 in year 3, etc.). Yet, the amount of borrowing capacity removed from the succeeding years declines as debts become less as principal is repaid each year. Interest is charged on the diminishing balance of the loan at an assumed rate of 6 percent.

The function of the intermediate-term loan activity is to finance investment in machinery and expansion of beef cow herd. Because of distinct characteristics of these two assets, two separate activities were included in the model. One of these activities, called "intermediate-term loan for machinery" (P25), was designed to finance purchase of farm machinery and equipment. A 10-year period for repayment of the principal was assumed. While this is somewhat longer than that cited by the typical farm finance textbook, it is reflective of the financial experiences of the study farm operator. For example, purchase of a dollar's worth of machinery through funds from a 10-year intermediate-term loan would require a cash payment of \$0.10 in each year. Interest on the loan at a rate of 7 percent requires a total cash outlay of \$0.17 at the end of the purchase year. Total cash payments decrease each year because interest payments on the outstanding balance decline year after year. This nature is clearly shown in the coefficients in the activity. Incidentally, a \$1 loan requires a borrowing capacity of \$1 in the year when the loan occurs. Requirements become proportionately smaller in following years as the loan is repaid. Maximum borrowing capacity for these type of loans was assumed to be 80 percent of total "intermediate" assets (cash, bonds and securities, livestock and machinery) minus total intermediate debts.

The structure of the financial activity for the cow purchase activity (P26) resembles that for the purchase of machinery. The only difference is the period of repayment. It was presumed that the operator would be able to extend the term of loans for breeding cows longer than that for the machinery investment. This does not mean the initial term is longer; it may be, and probably is, shorter. Still, as profitability induces the operator to borrow money to buy breeding cows, quality of the cow herd may be improved. The larger the cow herd expands the better the quality. Consequently, profitability together with the improving productivity of the cow herd is likely to enable him to renew a large part of the original face value of the note each year. A review of the study-farm operator's financial records also revealed support for this argument. Therefore, a 20-year maturity was assumed for financing investment in beef cows.

The short-term loan activity (P27) allows the manager to borrow money for operating purposes. Funds borrowed in each year were assumed to be repaid at the end of the same year. Interest is charged at the rate of 7 percent. Total cash payments associated with this loan are, therefore, \$1.07 for each \$1 borrowed. Maximum borrowing capacity for this loan is 90 percent of total liquid assets (cash, bonds and securities, livestock) minus total outstanding short-term debts.

Accounting Sector (P31, . . . , P35)

Three accounting activities were incorporated into the programming model. Two of these, (P28 and P29) are to account for annual cash outlays and the remaining one, P30, is to compute

annual overhead costs. Classification and computational procedures for these three expenses are discussed subsequently.

Transfer Sector (P31, . . . , P35)

Activities P31, P32, and P33 are provided to transfer feed grain and forages from year 1 to year 2, etc. Such activities recognize the fact that feed requirements for livestock in a specific year are partly supplied by production in the previous year(s). The quantity to be transferred is dependent upon the coefficients of feed requirements and the number of beef cattle to be raised.

Owing to the seasonality of cash flows, two cash transfer activities were written into the model. The cash transfer activity P34 serves to transfer cash savings from one season to another in a specific year. When cash receipts, generated from beef cattle sales in the summer and fall, are not exhausted, they are transferred to the cash receipts accounting row at the end of the production year. The other cash savings are transferred to the next production year by means of cash transfer activity II, P35.

The Constraints

Similar to activities, the constraint vector in the tableau for a particular year may be partitioned into several subvectors. The content of each subvector is discussed in the following subsections.

Resource Constraints

Twenty-two production constraints for each year, including labor, land, capital, feed, livestock (beef cows), and tobacco allotment are included in this group.

Total availability of the operator's labor for farming was assumed to equal 2,500 hours during a year. The same amount of annual labor supply was assumed for each of the two tenants. Each type of available labor was partitioned into calendar quarters. The exact quarterly labor distribution of the operator and tenants is shown in the "restriction level column" in the tableau. Farm operator labor may be supplemented, of course, by hired labor. This is made possible through the operation of labor hiring activities.

Land rows, R5 and R6, restrict production of field crops, forages and pasture to the acres of land available or purchased in the model. Initially, the study farm had 354 and 184 acres of tillable and non-tillable land, respectively.

The group "capital constraints" contains three components: money capital (R7), machine capital (R8) and buildings (R9). Both machinery and buildings are measured in dollar units. Initial availability of these three capital inputs can be readily determined from the tableau. It should be noted that the level of money capital (R7) available in any year subsequent to the first is zero it is generated only through the farm operations during the previous year(s). Decreasing values of machine capital and buildings, which may appear in production years subsequent to the first, are due to the depreciation of these assets. A supplement to any of these three capital items can be made by corresponding investment and borrowing activities.

The "feed constraints" sector includes four production rows and three storage rows (R10 through R16). Inclusion of feed production rows provides the model with the capacity to allocate available feeds among livestock activities and/or transfer feed surpluses to the next production year. The purpose of feed storage rows is to maintain the necessary quantity of feed for feeding livestock during the winter and early spring months. Pasture supply is computed in terms of dry matter.

The function of the "beef cow constraint" row (R17) and the tobacco allotment row (R18) is self-explanatory; namely, they are simply in recognition of "natural" or institutional restrictions on the production of calves and tobacco, respectively.

Investment Funds and Borrowing Capacity Constraints

Rows R19, R20, and R21 serve as bank accounts to provide a link between investment activities (P19-P23) and financing activities (P24-P27). Therefore, a zero value was given to these three rows in the "restriction level" column. To satisfy the need for investment funds through loans a coefficient is entered in the intersections of these rows and their corresponding investment activities. For example, machinery investment activity has a coefficient of 0.75 in the "investment funds for machinery" row, indicating that the purchase of \$1.00 worth of machinery needs \$0.75 from the loan (assuming a 25 percent downpayment).

Entries in the borrowing capacity rows R22, R23, and R24, reflect the maximum limits for loans available to the operator. Permissible maximum borrowing limits for the study farm were assumed to be determined from the following relations:

Maximum limit for short-term loans = $(0.9 \times \text{total liquid assets}) - (\text{total outstanding short-term debts})$

Maximum limit for intermediate-term loans = $(0.6 \times \text{total assets}) - (\text{total intermediate assets}) - (\text{total outstanding intermediate-term debts})$

Maximum limit for long-term loans = $(0.6 \times \text{total assets}) - (\text{total outstanding debts})$.

The term "liquid assets" refers to cash, bonds and securities, crop and livestock inventories while "intermediate assets" refers to the sum of liquid assets and the value of machinery on hand.

Restriction levels of these maximum borrowing limits are, as shown in the tableau, substantially lower in production years following the first. This is due to exclusion of the cash and livestock asset components which are unknown before running the model. The contribution of these two assets to the borrowing capacity in a particular year is entirely dependent upon the outcome of the farm operations during the previous production year (reflected in the simulation model).

Equality Constraints

The purpose of including equality constraints in the model is twofold. First, rows R25 and R26 serve as a device for forcing the model to sell calves and feeder cattle in the current year. This may be justified by the Kentucky farming practice that calves and feeder cattle carried over from the previous year usually are sold during the current year. Seldom are cattle fed to

"slaughter weights" in the central part of this state. Rows R27 and R27a insure that annual fixed cash expenses and fixed costs are paid. In this study, fixed cash expenses are divided into two parts. Basically, part one refers to those expenses which must be available at the beginning of the year, including family living expenses, machinery insurance and repair expenses. The second part of fixed cash expenses includes principal and interest payments, cash rent, property taxes and real estate taxes. All of these expenses are assumed to be paid at the end of each year. Computational procedures for these expense items are discussed later.

The inclusion of the fixed cost row (R28) in the model is to account for two implicit costs: (1) depreciation of machinery and buildings, and (2) capital charges against investment in land, livestock (beef cows) and depreciable assets. Interest is charged at 4 percent on land and 5 percent on the value of livestock and depreciable assets. This capital charge has been, in recent years, regarded as an approximate amount of interest earned from the investment in relatively "risk-free" securities [Kentucky Farm Analysis Groups, 1970, p. 5].

The Objective Function

The objective function, maximization of net returns to the unpaid labor and management for an entire 5-year planning period, was selected primarily on the basis of two considerations. To expand the farm business, maximum net returns would certainly provide the operator with the greatest amount of internal capital. Even though expansion funds may be obtained from loans, the operator's action is often hindered by capital rationing—high interest rates. Moreover, maximization of net returns is one of important economic factors motivating the operator to expand his farm business.

Some coefficients (denoted by symbols) in the objective function parameters are estimated from the expectation model at the time the long-run plan is made. Therefore, the solution of the programming model should be regarded as "expected" rather than "realized" maximum net returns for the entire planning period.

Appendix A (Cont'd)

Constraint	Code Unit	Activity →	Hired Labor	Land Purchase (Tillable)	Land Purchase (Non-tillable)	Mech. Invest.	Blde. Invest.	Beef Cow Purchase	Borrow Capital (Long)	Borrow Capital for Mech.	Borrow Capital for Cows	Borrow Capital (Short)	Fixed Cash Expenses II	Fixed Cash Expenses I	Corn Silage Trans.	Corn Silage Trans. I	Cash Trans. II	Cash Trans. I				
		Restriction Level	ID	IP18	IP19	IP20	IP21	IP22	IP23	IP24	IP25	IP26	IP27	IP28	IP29	IP30	IP31	IP32	IP33	IP34	IP35	
	Objective Function →	N																				
Operator's Labor	281 Hr.	600	L																			
	282 Hr.	675	L																			
	283 Hr.	675	L																			
Tenant's Labor	284 Hr.	600	L																			
	281a Hr.	1200	L																			
	282a Hr.	1350	L																			
	283a Hr.	1500	L																			
	284a Hr.	1200	L																			
	285 Ac.	354	L																			
Total Tillable Land	286 Ac.	184	L		-20.0																	
Permanent Pasture Land	287 DoI.	0	L																			
Machine Capital	288 DoI.	0	L																			
Buildings	289 DoI.	3,733	L																			
Corn Production	290 Bu.	0	L																			
Corn Silage Production	2812 Ton	0	L																			
Corn Silage Storage	2813 Ton	0	L																			
Hay Production	2814 Ton	0	L																			
Hay Storage	2815 Ton	0	L																			
Pasture Production	2816 Ton	71	L																			
Pasture Storage	2817 Ton	0	L																			
Tobacco Allotment	2818 Ac.	9.67	L																			
Invest. Funds for Mach. and Land	2819 DoI.	0	L																			
Invest. Funds for Cows	2820 DoI.	0	L																			
Borrow Capacity (Short)	2821 DoI.	15,910	L																			
Borrow Capacity (Long)	2822 DoI.	18,732	L																			
Borrow Capacity (Long)	2823 DoI.	18,414	L		-6000.0	-1800.0					0.90	0.95										
Calif. Inventory	2824 DoI.	0	E																			
Feeder Cattle Inventory	2825 An.	0	E																			
Fixed Cash Expenses I	2826 An.	3,123	E																			
Fixed Cash Expenses II	2827a DoI.	10,725	E																			
Fixed Cash Expenses II	2827b DoI.	2,950	E																			
Cash Receipts I	2828 DoI.	0	L																			
Cash Receipts II	2829a DoI.	0	L																			
	2829b DoI.	0	L																			

-1.0

-1.0

-1.0

-1.0

-1.0

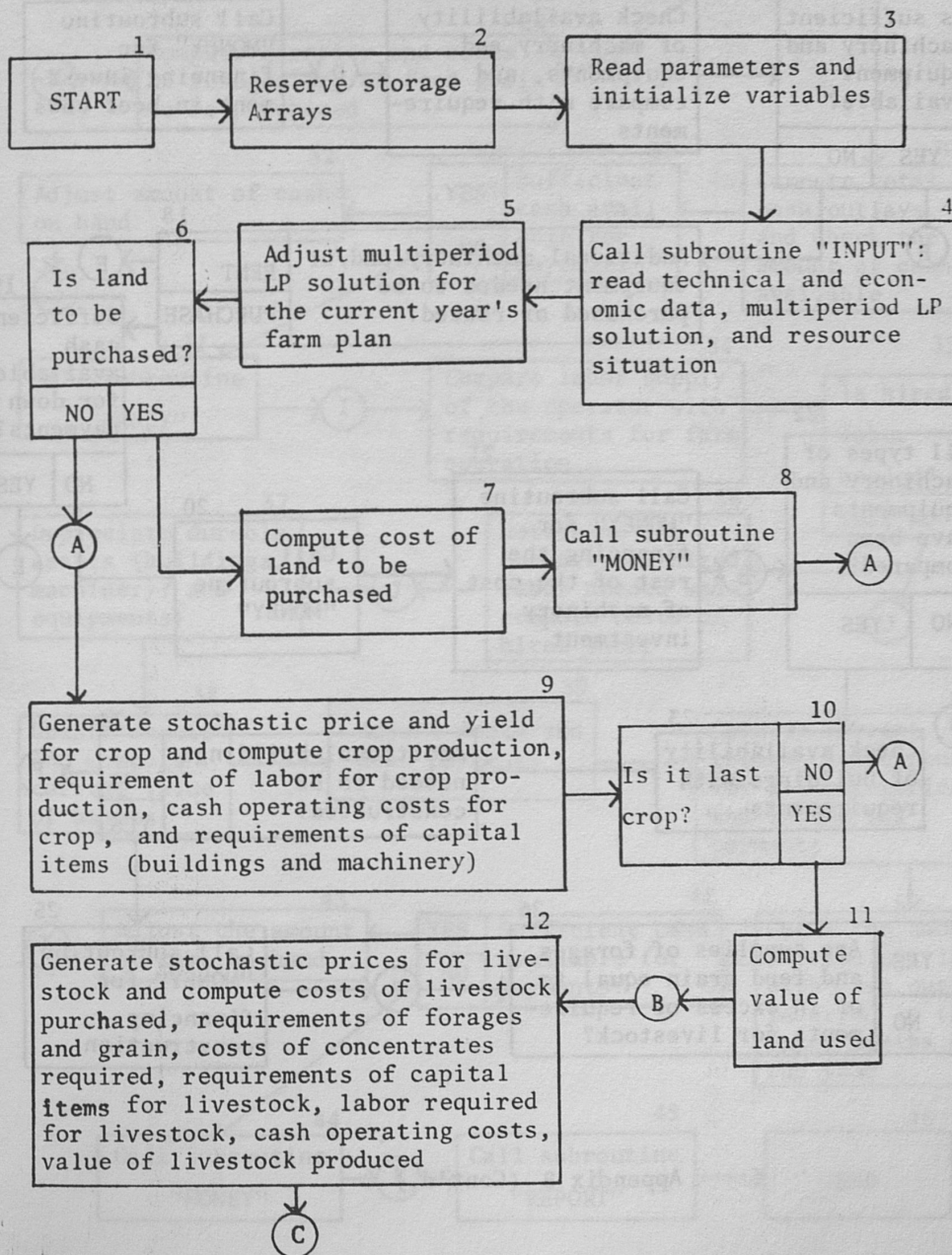
-1.0

-1.0

-1.0

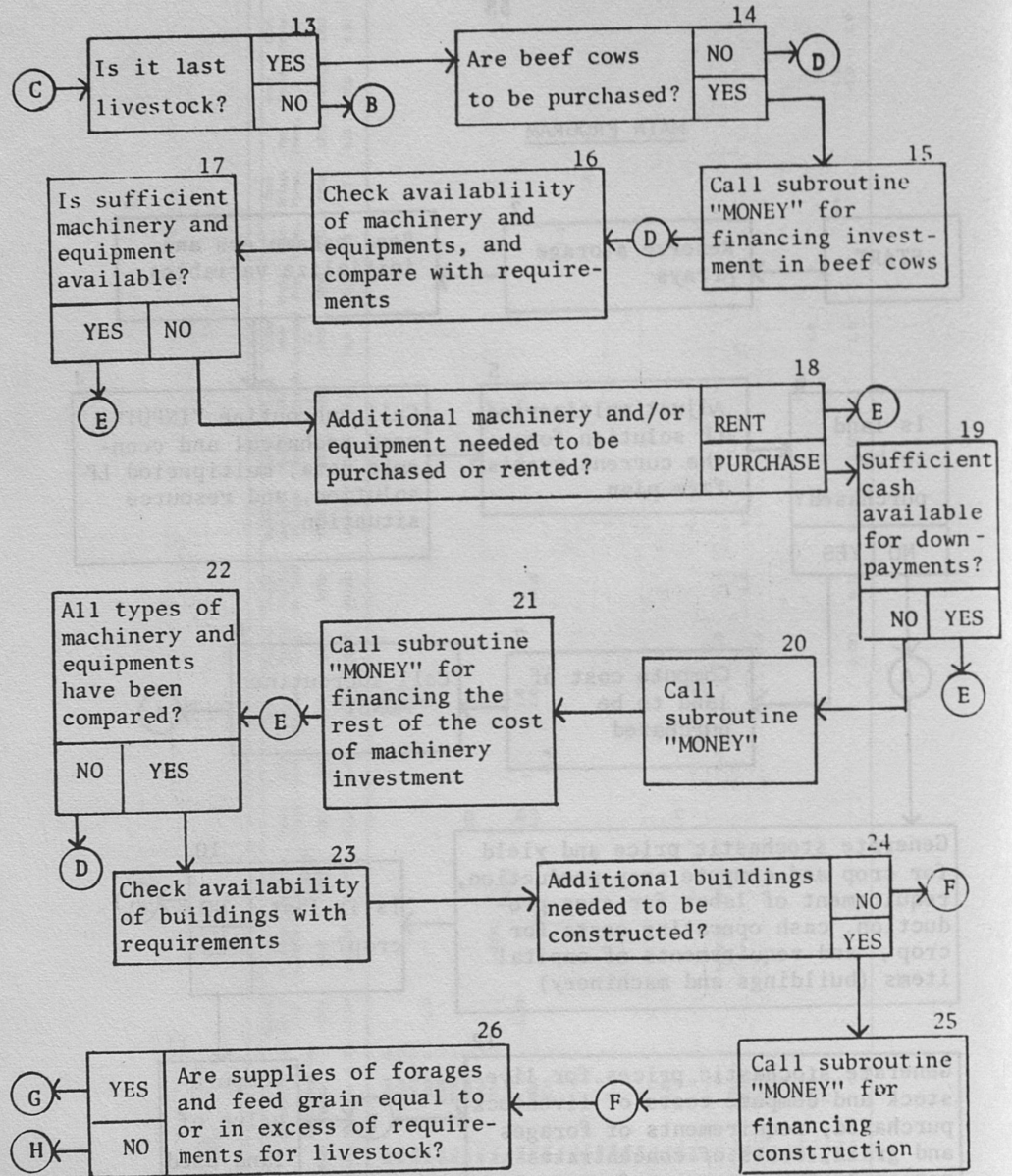
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MAIN PROGRAM



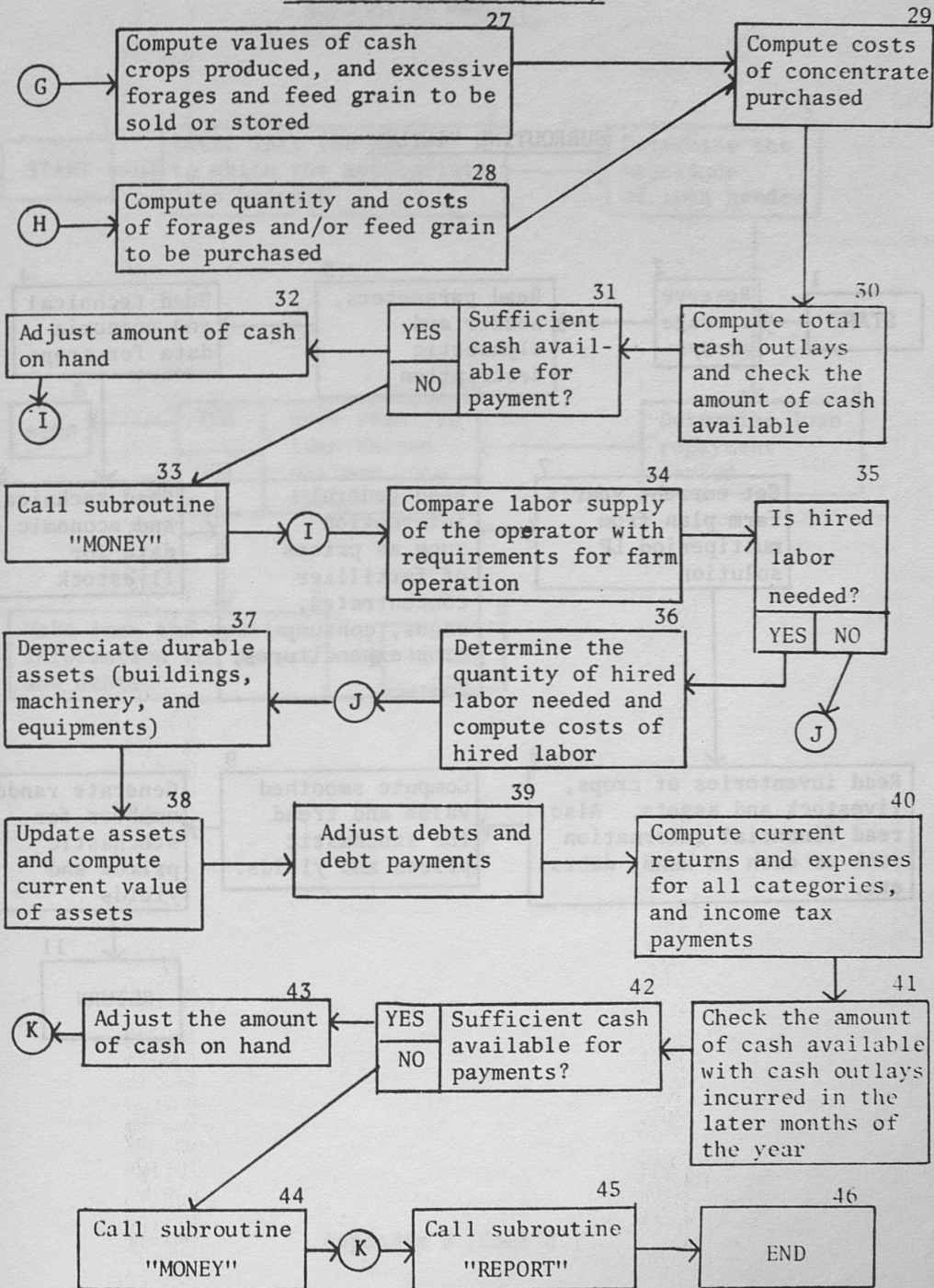
Appendix B - Flow Chart for the Simulation Model of the Farm Operation

MAIN PROGRAM (Continued)



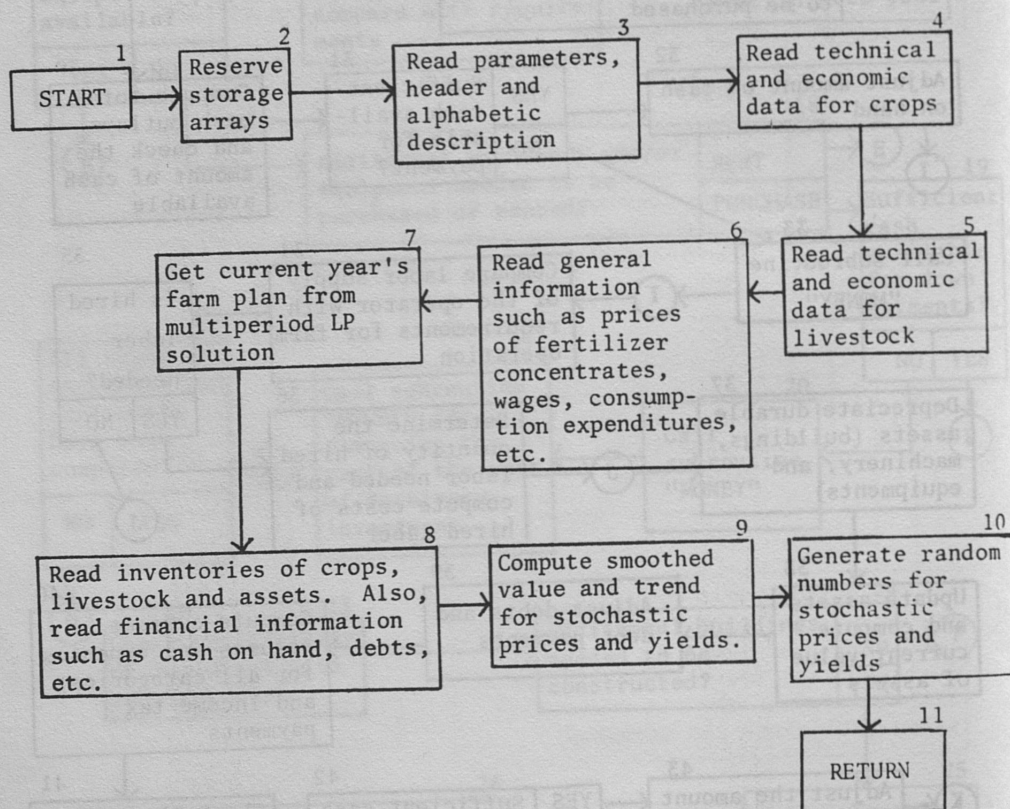
Appendix B (Cont'd.)

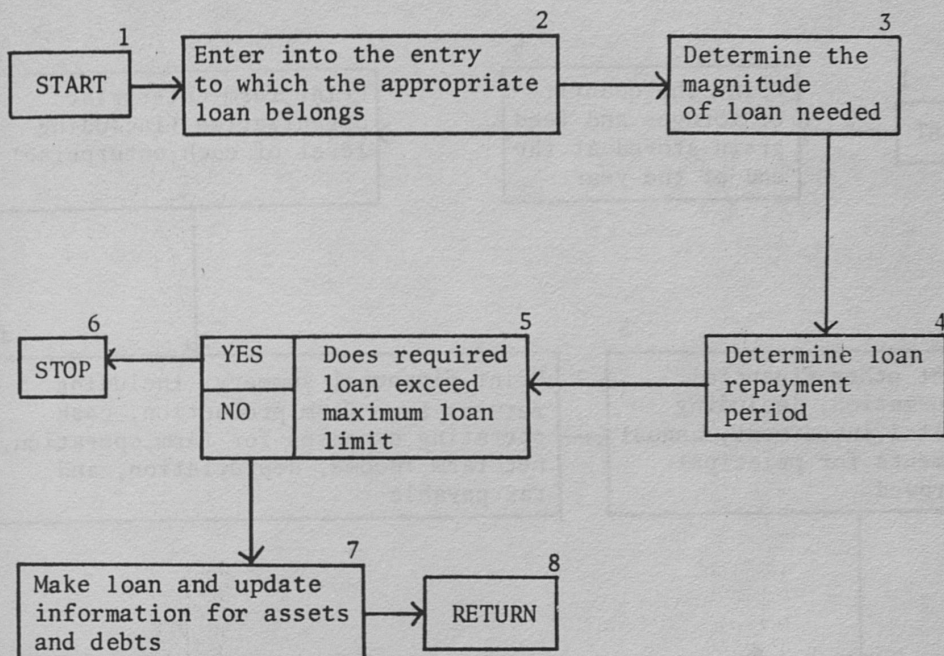
MAIN PROGRAM (Continued)



Appendix B (Cont'd.)

SUBROUTINE "INPUT"



SUBROUTINE "MONEY"

SUBROUTINE "REPORT"